- $\mathrm{Cov}[aX, bY] = ab\mathrm{Cov}[X, Y]$
- $\mathrm{Cov}[X + a, Y + b] = \mathrm{Cov}[X, Y]$
- $\mathrm{Cov}\left[\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right] = \sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{Cov}[X_i, Y_j]$

Correlation

$$\rho[X, Y] = \frac{\mathrm{Cov}[X, Y]}{\sqrt{\mathbb{V}[X]\,\mathbb{V}[Y]}}$$

Independence

$$X \perp\!\!\!\perp Y \implies \rho[X, Y] = 0 \iff \mathrm{Cov}[X, Y] = 0 \iff \mathbb{E}[XY] = \mathbb{E}[X]\,\mathbb{E}[Y]$$

Sample variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$$

Conditional variance

- $\mathbb{V}[Y \mid X] = \mathbb{E}\left[(Y - \mathbb{E}[Y \mid X])^2 \mid X\right] = \mathbb{E}\left[Y^2 \mid X\right] - \mathbb{E}[Y \mid X]^2$
- $\mathbb{V}[Y] = \mathbb{E}[\mathbb{V}[Y \mid X]] + \mathbb{V}[\mathbb{E}[Y \mid X]]$

6 Inequalities

Cauchy-Schwarz

$$\mathbb{E}[XY]^2 \le \mathbb{E}\left[X^2\right]\mathbb{E}\left[Y^2\right]$$

Markov

$$\mathbb{P}[\varphi(X) \ge t] \le \frac{\mathbb{E}[\varphi(X)]}{t}$$

Chebyshev

$$\mathbb{P}[|X - \mathbb{E}[X]| \ge t] \le \frac{\mathbb{V}[X]}{t^2}$$

Chernoff

$$\mathbb{P}[X \ge (1+\delta)\mu] \le \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right) \quad \delta > -1$$

Hoeffding

$$X_1, \ldots, X_n \text{ independent } \wedge\ \mathbb{P}[X_i \in [a_i, b_i]] = 1\ \wedge\ 1 \le i \le n$$

$$\mathbb{P}\left[\bar{X} - \mathbb{E}\left[\bar{X}\right] \ge t\right] \le e^{-2nt^2} \quad t > 0$$

$$\mathbb{P}\left[|\bar{X} - \mathbb{E}\left[\bar{X}\right]| \ge t\right] \le 2\exp\left\{-\frac{2n^2t^2}{\sum_{i=1}^{n}(b_i - a_i)^2}\right\} \quad t > 0$$

Jensen

$$\mathbb{E}[\varphi(X)] \ge \varphi(\mathbb{E}[X]) \quad \varphi \text{ convex}$$

7 Distribution Relationships

Binomial

- $X_i \sim \mathrm{Bern}(p) \implies \sum_{i=1}^{n} X_i \sim \mathrm{Bin}(n, p)$
- $X \sim \mathrm{Bin}(n, p), Y \sim \mathrm{Bin}(m, p) \implies X + Y \sim \mathrm{Bin}(n + m, p)$
- $\lim_{n\to\infty} \mathrm{Bin}(n, p) = \mathrm{Po}(np) \qquad (n \text{ large, } p \text{ small})$
- $\lim_{n\to\infty} \mathrm{Bin}(n, p) = \mathcal{N}(np, np(1-p)) \qquad (n \text{ large, } p \text{ far from 0 and 1})$

Negative Binomial

- $X \sim \mathrm{NBin}(1, p) = \mathrm{Geo}(p)$
- $X \sim \mathrm{NBin}(r, p) = \sum_{i=1}^{r} \mathrm{Geo}(p)$
- $X_i \sim \mathrm{NBin}(r_i, p) \implies \sum X_i \sim \mathrm{NBin}(\sum r_i, p)$
- $X \sim \mathrm{NBin}(r, p).\ Y \sim \mathrm{Bin}(s + r, p) \implies \mathbb{P}[X \le s] = \mathbb{P}[Y \ge r]$

Poisson

- $X_i \sim \mathrm{Po}(\lambda_i) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_{i=1}^{n} X_i \sim \mathrm{Po}\left(\sum_{i=1}^{n} \lambda_i\right)$
- $X_i \sim \mathrm{Po}(\lambda_i) \wedge X_i \perp\!\!\!\perp X_j \implies X_i \left| \sum_{j=1}^{n} X_j \right. \sim \mathrm{Bin}\left(\sum_{j=1}^{n} X_j, \frac{\lambda_i}{\sum_{j=1}^{n} \lambda_j}\right)$

Exponential

- $X_i \sim \mathrm{Exp}(\beta) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_{i=1}^{n} X_i \sim \mathrm{Gamma}(n, \beta)$
- Memoryless property: $\mathbb{P}[X > x + y \mid X > y] = \mathbb{P}[X > x]$

Normal

- $X \sim \mathcal{N}\left(\mu, \sigma^2\right) \implies \left(\frac{X-\mu}{\sigma}\right) \sim \mathcal{N}(0, 1)$
- $X \sim \mathcal{N}\left(\mu, \sigma^2\right) \wedge Z = aX + b \implies Z \sim \mathcal{N}\left(a\mu + b, a^2\sigma^2\right)$
- $X_i \sim \mathcal{N}\left(\mu_i, \sigma_i^2\right) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_i X_i \sim \mathcal{N}\left(\sum_i \mu_i, \sum_i \sigma_i^2\right)$
- $\mathbb{P}[a < X \le b] = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$
- $\Phi(-x) = 1 - \Phi(x) \qquad \phi'(x) = -x\phi(x) \qquad \phi''(x) = (x^2 - 1)\phi(x)$
- Upper quantile of $\mathcal{N}(0, 1)$: $z_\alpha = \Phi^{-1}(1 - \alpha)$

Gamma

- $X \sim \mathrm{Gamma}(\alpha, \beta) \iff X/\beta \sim \mathrm{Gamma}(\alpha, 1)$
- $\mathrm{Gamma}(\alpha, \beta) \sim \sum_{i=1}^{\alpha} \mathrm{Exp}(\beta)$
- $X_i \sim \mathrm{Gamma}(\alpha_i, \beta) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_i X_i \sim \mathrm{Gamma}(\sum_i \alpha_i, \beta)$