Shoe for Sale

Nude High Cut Shoe

- 10cm Natural Leather Stacked Heel.
- Multiple Strap Detail With Gold Western Buckles & Functional Zip.
- Leather Lining & Sock
- Unique

The original price of a pair was $500.
Now the last piece for $5 only.

Combinatorial auctions

- A combinatorial auction sells multiple objects semitanuously.
- Bidders can place bids on combinations of items in bundles.
- Bidder's valuation on a bundle may be different from the sum of the valuations of all the items in the bundle.
    - Complementary: the value of a combination of items is worth more than the sum of the values of the separate items.
    - Substitutable: the value of a combination of items is less than the sum of the values of the separate items.

The model of combinatorial auctions

$E = (N \cup \{0\}, X, \{v_i\}_{i \in N})$ is a combinatorial auction if

- $N = \{1, 2, \cdots, n\}$ is the set of buyers
- 0 represents the seller
- X is the set of items
- $v_i : 2^X \to \mathbb{Z}^+$ the buyer i's value function

Example:

$N = \{1, 2\}$, $X = \{a, b\}$.
$v_1(\emptyset) = 0,\ v_1(\{a\}) = v_1(\{b\}) = v_1(\{a, b\}) = 1.$
$v_2(\emptyset) = 0,\ v_2(\{a\}) = v_2(\{b\}) = 1,\ v_2(\{a, b\}) = 3.$

Question: How to allocate the items to the buyers so that each item goes to the buyer who gives it the highest value?

Efficient allocations

- Allocation: $\pi : N \cup \{0\} \to 2^X$ such that
  - $\pi(i) \cap \pi(j) = \emptyset$ for any $i \neq j$.
  - $\bigcup_{i \in N \cup \{0\}} \pi(i) = X$.

  which allocate all the items to the buyers, each buyer can have a bundle but one item can only be allocated to at most one buyer.
- Efficient allocation π^*: $\pi^*(0) = \emptyset$ and for every allocation π of X,

$$\sum_{i \in N} v_i(\pi^*(i)) \geq \sum_{i \in N} v_i(\pi(i))$$

Question: How to find an efficient allocation?

Walrasian equilibria

- Price vector $\mathbf{p}$: assign a non-negative real number to each item in X.
- Demand correspondence:

$$D_i(\mathbf{p}) = \arg\max_{A \subseteq X} (V_i(A) - \sum_{a \in A} p_a)$$

representing all the bundles that give i the highest utility based on the current market price. For instance, if $\mathbf{p} = (0.5, 0.5)$,

$$D_1(\mathbf{p}) = \{\{a\}, \{b\}\}.\ D_2(\mathbf{p}) = \{\{a, b\}\}$$

- Walrasian equilibrium $(\mathbf{p}, \pi)$: $\mathbf{p}$ is a price vector and π is an allocation of X such that $\pi(0) = \emptyset$ and $\pi(i) \in D_i(\mathbf{p})$ for all $i \in N$.
- Any Walrasian equilibrium determines an efficient allocation.

Question: How to find a Walrasian equilibrium?