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How to Think about Strategic Games

CHAPTER 1 GAVE SOME simple examples of strategic games and strategic thinking. In this chapter, we begin a more systematic and analytical approach to the subject. We choose some crucial conceptual categories or dimensions, each of which has a dichotomy of types of strategic interactions. For example, one such dimension concerns the timing of the players' actions, and the two pure types are games where the players act in strict turns (sequential moves) and where they act at the same time (simultaneous moves). We consider some matters that arise in thinking about each pure type in this dichotomy, as well as in similar dichotomies with respect to other matters, such as whether the game is played only once or repeatedly and what the players know about each other.

In Chapters 3–7, we will examine each of these categories or dimensions in more detail; in Chapters 8–17, we will show how the analysis can be used in several contexts. Of course, most actual applications are not of a pure type but rather a mixture. Moreover, in each application, two or more of the categories have some relevance. The lessons learned from the study of the pure types must therefore be combined in appropriate ways. We will show how to do this by using the context of our applications.

In this chapter, we state some basic concepts and terminology—such as strategies, payoffs, and equilibrium—that are used in the analysis and briefly describe solution methods. We also provide a brief discussion of the uses of game theory and an overview of the structure of the remainder of the book.

1 DECISIONS VERSUS GAMES

When a person (or team or firm or government) decides how to act in dealings with other people (or teams or firms or governments), there must be some cross-effect of their actions; what one does must affect the outcome for the other. When George Pickett (of Pickett's Charge at the battle of Gettysburg) was asked to explain the Confederacy's defeat in the Civil War, he responded, "I think the Yankees had something to do with it."[1]

For the interaction to become a strategic game, however, we need something more—namely, the participants' mutual awareness of this cross-effect. What the other person does affects you; if you know this, you can react to his actions, or take advance actions to forestall the bad effects his future actions may have on you and to facilitate any good effects, or even take advance actions so as to alter his future reactions to your advantage. If you know that the other person knows that what you do affects him, you know that he will be taking similar actions. And so on. It is this mutual awareness of the cross-effects of actions and the actions taken as a result of this awareness that constitute the most interesting aspects of strategy.

This distinction is captured by reserving the label **strategic games** (or sometimes just **games**, because we are not concerned with other types of games, such as those of pure chance or pure skill) for interactions between mutually aware players and **decisions** for action situations where each person can choose without concern for reaction or response from others. If Robert E. Lee (who ordered Pickett to lead the ill-fated Pickett's Charge) had thought that the Yankees had been weakened by his earlier artillery barrage to the point that they no longer had any ability to resist, his choice to attack would have been a decision; if he was aware that the Yankees were prepared and waiting for his attack, then the choice became a part of a (deadly) game. The simple rule is that unless there are two or more players, each of whom responds to what others do (or what each thinks the others might do), it is not a game.

Strategic games arise most prominently in head-to-head confrontations of two participants: the arms race between the United States and the Soviet Union from the 1950s through the 1980s; wage negotiations between General Motors and the United Auto Workers; or a Super Bowl matchup between two "pirates," the Tampa Bay Buccaneers and the Oakland Raiders. In contrast, interactions among a large number of participants seem less susceptible to the issues raised by mutual awareness. Because each farmer's output is an insignificant part of the whole nation's or the world's output, the decision of one farmer to grow

[1] James M. McPherson, "American Victory, American Defeat," in *Why the Confederacy Lost*, ed. Gabor S. Boritt (New York: Oxford University Press, 1993), p. 19.

more or less corn has almost no effect on the market price, and not much appears to hinge on thinking of agriculture as a strategic game. This was indeed the view prevalent in economics for many years. A few confrontations between large companies—as in the U.S. auto market, which was once dominated by GM, Ford, and Chrysler—were usefully thought of as strategic games, but most economic interactions were supposed to be governed by the impersonal forces of supply and demand.

In fact, game theory has a much greater scope. Many situations that start out as impersonal markets with thousands of participants turn into strategic interactions of two or just a few. This happens for one of two broad classes of reasons—mutual commitments or private information.

Consider commitment first. When you are contemplating building a house, you can choose one of several dozen contractors in your area; the contractor can similarly choose from several potential customers. There appears to be an impersonal market. Once each side has made a choice, however, the customer pays an initial installment, and the builder buys some materials for the plan of this particular house. The two become tied to each other, separately from the market. Their relationship becomes *bilateral*. The builder can try to get away with a somewhat sloppy job or can procrastinate, and the client can try to delay payment of the next installment. Strategy enters the picture. Their initial contract in the market has to anticipate their individual incentives in the game to come and specify a schedule of installments of payments that are tied to successive steps in the completion of the project. Even then, some adjustments have to be made after the fact, and these adjustments bring in new elements of strategy.

Next, consider private information. Thousands of farmers seek to borrow money for their initial expenditures on machinery, seed, fertilizer, and so forth, and hundreds of banks exist to lend to them. Yet the market for such loans is not impersonal. A borrower with good farming skills who puts in a lot of effort will be more likely to be successful and will repay the loan; a less-skilled or lazy borrower may fail at farming and default on the loan. The risk of default is highly personalized. It is not a vague entity called "the market" that defaults, but individual borrowers who do so. Therefore each bank will have to view its lending relation with each individual borrower as a separate game. It will seek collateral from each borrower or will investigate each borrower's creditworthiness. The farmer will look for ways to convince the bank of his quality as a borrower; the bank will look for effective ways to ascertain the truth of the farmer's claims.

Similarly, an insurance company will make some efforts to determine the health of individual applicants and will check for any evidence of arson when a claim for a fire is made; an employer will inquire into the qualifications of individual employees and monitor their performance. More generally, when participants in a transaction possess some private information bearing on the outcome, each bilateral deal becomes a game of strategy, even though the larger picture may have thousands of very similar deals going on.

To sum up, when each participant is significant in the interaction, either because each is a large player to start with or because commitments or private information narrow the scope of the relationship to a point where each is an important player *within* the relationship, we must think of the interaction as a strategic game. Such situations are the rule rather than the exception in business, in politics, and even in social interactions. Therefore, the study of strategic games forms an important part of all fields that analyze these matters.

2 CLASSIFYING GAMES

Games of strategy arise in many different contexts and accordingly have many different features that require study. This task can be simplified by grouping these features into a few categories or dimensions, along each of which we can identify two pure types of games and then recognize any actual game as a mixture of the pure types. We develop this classification by asking a few questions that will be pertinent for thinking about the actual game that you are playing or studying.

A. Are the Moves in the Game Sequential or Simultaneous?

Moves in chess are sequential: White moves first, then Black, then White again, and so on. In contrast, participants in an auction for an oil-drilling lease or a part of the airwave spectrum make their bids simultaneously, in ignorance of competitors' bids. Most actual games combine aspects of both. In a race to research and develop a new product, the firms act simultaneously, but each competitor has partial information about the others' progress and can respond. During one play in football, the opposing offensive and defensive coaches simultaneously send out teams with the expectation of carrying out certain plays, but after seeing how the defense has set up, the quarterback can change the play at the line of scrimmage or call a time-out so that the coach can change the play.

The distinction between **sequential** and **simultaneous moves** is important because the two types of games require different types of interactive thinking. In a sequential-move game, each player must think: If I do this, how will my opponent react? Your current move is governed by your calculation of its *future* consequences. With simultaneous moves, you have the trickier task of trying to figure out what your opponent is going to do *right now*. But you must recognize that, in making his own calculation, your opponent is also trying to figure out your current move, while at the same time recognizing that you are doing the same with him. . . . Both of you have to think your way out of this circle.

In the next three chapters, we will study the two pure cases. In Chapter 3, we examine sequential-move games, where you must look ahead to act now;

in Chapters 4 and 5, the subject is simultaneous-move games, where you must square the circle of "He thinks that I think that he thinks . . ." In each case, we will devise some simple tools for such thinking—trees and payoff tables—and obtain some simple rules to guide actions.

The study of sequential games also tells us when it is an advantage to move first and when it is an advantage to move second. Roughly speaking, this depends on the relative importance of commitment and flexibility in the game in question. For example, the game of economic competition among rival firms in a market has a first-mover advantage if one firm, by making a firm commitment to compete aggressively, can get its rivals to back off. But, in political competition, a candidate who has taken a firm stand on an issue may give his rivals a clear focus for their attack ads, and the game has a second-mover advantage.

Knowledge of the balance of these considerations can also help you devise ways to manipulate the order of moves to your own advantage. That in turn leads to the study of strategic moves, such as threats and promises, which we will take up in Chapter 9.

B. Are the Players' Interests in Total Conflict or Is There Some Commonality?

In simple games such as chess or football, there is a winner and a loser. One player's gain is the other's loss. Similarly, in gambling games, one player's winnings are the others' losses, so the total is 0. This is why such situations are called **zero-sum games**. More generally, the idea is that the players' interests are in complete conflict. Such conflict arises when players are dividing up any fixed amount of possible gain, whether it be measured in yards, dollars, acres, or scoops of ice cream. Because the available gain need not always be exactly 0, the term **constant-sum game** is often substituted for zero-sum game; we will use the two terms interchangeably.

Most economic and social games are not zero-sum. Trade, or economic activity more generally, offers scope for deals that benefit everyone. Joint ventures can combine the participants' different skills and generate synergy to produce more than the sum of what they could have produced separately. But the interests are not completely aligned either; the partners can cooperate to create a larger total pie, but they will clash when it comes to deciding how to split this pie among them.

Even wars and strikes are not zero-sum games. A nuclear war is the most striking example of a situation where there can only be losers, but the concept is far older. Pyrrhus, the king of Epirus, defeated the Romans at Heraclea in 280 B.C. but at such great cost to his own army that he exclaimed, "Another such victory and we are lost!" Hence the phrase "Pyrrhic victory." In the 1980s, at the height of the frenzy of business takeovers, the battles among rival bidders led to such costly escalation that the successful bidder's victory was often similarly Pyrrhic.

Most games in reality have this tension between conflict and cooperation, and many of the most interesting analyses in game theory come from the need to handle it. The players' attempts to resolve their conflict—distribution of territory or profit—are influenced by the knowledge that, if they fail to agree, the outcome will be bad for all of them. One side's threat of a war or a strike is its attempt to frighten the other side into conceding its demands.

Even when a game is constant-sum for all players, if it has three (or more) players, we have the possibility that two of them will cooperate at the expense of the third; this leads to the study of alliances and coalitions. We will examine and illustrate these ideas later, especially in Chapter 17 on bargaining.

C. Is the Game Played Once or Repeatedly, and with the Same or Changing Opponents?

A game played just once is in some respects simpler and in others more complicated than one that includes many interactions. You can think about a one-shot game without worrying about its repercussions on other games you might play in the future against the same person or against others who might hear of your actions in this one. Therefore actions in one-shot games are more likely to be unscrupulous or ruthless. For example, an automobile repair shop is much more likely to overcharge a passing motorist than a regular customer.

In one-shot encounters, each player doesn't know much about the others; for example, what their capabilities and priorities are, whether they are good at calculating their best strategies or have any weaknesses that can be exploited, and so on. Therefore in one-shot games, secrecy or surprise is likely to be an important component of good strategy.

Games with ongoing relationships require the opposite considerations. You have an opportunity to build a reputation (for toughness, fairness, honesty, reliability, and so forth, depending on the circumstances) and to find out more about your opponent. The players together can better exploit mutually beneficial prospects by arranging to divide the spoils over time (taking turns to "win") or to punish a cheater in future plays (an eye for an eye or tit-for-tat). We will consider these possibilities in Chapter 10 on the prisoners' dilemma.

More generally, a game may be zero-sum in the short run but have scope for mutual benefit in the long run. For example, each football team likes to win, but they all recognize that close competition generates more spectator interest, which benefits all teams in the long run. That is why they agree to a drafting scheme where teams get to pick players in reverse order of their current standing, thereby reducing the inequality of talent. In long-distance races, the runners or cyclists often develop a lot of cooperation; two or more of them can help one another by taking turns following in one another's slipstream. But near the end of the race, the cooperation collapses as all of them dash for the finish line.

Here is a useful rule of thumb for your own strategic actions in life. In a game that has some conflict and some scope for cooperation, you will often think up a great strategy for winning big and grinding a rival into the dust but have a nagging worry at the back of your mind that you are behaving like the worst 1980s yuppie. In such a situation, the chances are that the game has a repeated or ongoing aspect that you have overlooked. Your aggressive strategy may gain you a short-run advantage, but its long-run side effects will cost you even more. Therefore, you should dig deeper and recognize the cooperative element and then alter your strategy accordingly. You will be surprised how often niceness, integrity, and the golden rule of doing to others as you would have them do to you turn out to be not just old nostrums, but good strategies as well when you consider the whole complex of games that you will be playing in the course of your life.

D. Do the Players Have Full or Equal Information?

In chess, each player knows exactly the current situation and all the moves that led to it, and each knows that the other aims to win. This situation is exceptional; in most other games, the players face some limitation of information. Such limitations come in two kinds. First, a player may not know all the information that is pertinent for the choice that he has to make at every point in the game. This type of information problem arises because of the player's uncertainty about relevant variables, both internal and external to the game. For example, he may be uncertain about external circumstances, such as the weekend weather or the quality of a product he wishes to purchase; we call this situation one of **external uncertainty**. Or he may be uncertain about exactly what moves his opponent has made in the past or is making at the same time he makes his own move; we call this **strategic uncertainty**. If a game has neither external nor strategic uncertainty, we say that the game is one of **perfect information**; otherwise the game has **imperfect information**. We will give a more precise technical definition of perfect information in Chapter 6, Section 3.A, after we have introduced the concept of an information set. We will develop the theory of games with imperfect information (uncertainty) in three future chapters. In Chapter 4, we discuss games with contemporaneous (simultaneous) actions, which entail strategic uncertainty, and we analyze methods for making choices under external uncertainty in Chapter 8 and its appendix.

Trickier strategic situations arise when one player knows more than another does; they are called situations of *incomplete* or, better, **asymmetric information**. In such situations, the players' attempts to infer, conceal, or sometimes convey their private information become an important part of the game and the strategies. In bridge or poker, each player has only partial knowledge of the cards held by the others. Their actions (bidding and play in bridge, the number of cards taken and the betting behavior in poker) give information to opponents. Each player tries to manipulate his actions to mislead the opponents (and, in bridge, to inform his partner truthfully), but in doing so each must be aware that the opponents know this and that they will use strategic thinking to interpret that player's actions.

You may think that if you have superior information, you should always conceal it from other players. But that is not true. For example, suppose you are the CEO of a pharmaceutical firm that is engaged in an R&D competition to develop a new drug. If your scientists make a discovery that is a big step forward, you may want to let your competitors know, in the hope that they will give up their own searches and you won't face any future competition. In war, each side wants to keep its tactics and troop deployments secret; but, in diplomacy, if your intentions are peaceful, then you desperately want other countries to know and believe this fact.

The general principle here is that you want to release your information selectively. You want to reveal the good information (the kind that will draw responses from the other players that work to your advantage) and conceal the bad (the kind that may work to your disadvantage).

This raises a problem. Your opponents in a strategic game are purposive, rational players, and they know that you are, too. They will recognize your incentive to exaggerate or even to lie. Therefore, they are not going to accept your unsupported declarations about your progress or capabilities. They can be convinced only by objective evidence or by actions that are credible proof of your information. Such actions on the part of the more-informed player are called **signals**, and strategies that use them are called **signaling**. Conversely, the less-informed party can create situations in which the more-informed player will have to take some action that credibly reveals his information; such strategies are called **screening**, and the methods they use are called **screening devices**. The word *screening* is used here in the sense of testing in order to sift or separate, not in the sense of concealing.

Sometimes the same action may be used as a signal by the informed player or as a screening device by the uninformed player. Recall that in the dating game in Section 2.F of Chapter 1, the woman was screening the man to test his commitment to their relationship, and her suggestion that the pair give up one of their two rent-controlled apartments was the screening device. If the man had been committed to the relationship, he might have acted first and volunteered to give up his apartment; this action would have been a signal of his commitment.

Now we see how, when different players have different information, the manipulation of information itself becomes a game, perhaps more important than the game that will be played after the information stage. Such information games are ubiquitous, and playing them well is essential for success in life. We will study more games of this kind in greater detail in Chapter 8 and also in Chapter 13.

E. Are the Rules of the Game Fixed or Manipulable?

The rules of chess, card games, or sports are given, and every player must follow them, no matter how arbitrary or strange they seem. But in games of business, politics, and ordinary life, the players can make their own rules to a greater or lesser extent. For example, in the home, parents constantly try to make the rules, and children constantly look for ways to manipulate or circumvent those rules. In legislatures, rules for the progress of a bill (including the order in which amendments and main motions are voted on) are fixed, but the game that sets the agenda—which amendments are brought to a vote first—can be manipulated. This is where political skill and power have the most scope, and we will address these matters in detail in Chapter 15.

In such situations, the real game is the "pregame" where rules are made, and your strategic skill must be deployed at that point. The actual playing out of the subsequent game can be more mechanical; you could even delegate it to someone else. However, if you "sleep" through the pregame, you might find that you have lost the game before it ever began. For many years, American firms ignored the rise of foreign competition in just this way and ultimately paid the price. But some entrepreneurs, such as oil magnate John D. Rockefeller Sr., adopted the strategy of limiting their participation to games in which they could also participate in making the rules.[2]

The distinction between changing rules and acting within the chosen rules will be most important for us in our study of strategic moves, such as threats and promises. Questions of how you can make your own threats and promises credible or how you can reduce the credibility of your opponent's threats basically have to do with a pregame that entails manipulating the rules of the subsequent game in which the promises or threats may have to be carried out. More generally, the strategic moves that we will study in Chapter 9 are essentially ploys for such manipulation of rules.

But if the pregame of rule manipulation is the real game, what fixes the rules of the pregame? Usually these pregame rules depend on some hard facts related to the players' innate abilities. In business competition, one firm can take preemptive actions that alter subsequent games between it and its rivals; for example, it can expand its factory or advertise in a way that twists the results of subsequent price competition more favorably to itself. Which firm can do this best or most easily depends on which one has the managerial or organizational resources to make the investments or to launch the advertising campaigns.

Players may also be unsure of their rivals' abilities. This often makes the pregame one of incomplete or asymmetric information, requiring more subtle strategies and occasionally resulting in some big surprises. We will comment on all these matters in the appropriate places in the chapters that follow.

[2] For more on the methods used in Rockefeller's rise to power, see Ron Chernow, *Titan* (New York: Random House, 1998).

F. Are Agreements to Cooperate Enforceable?

We saw that most strategic interactions consist of a mixture of conflict and common interest. Then there is a case to be made that all participants should get together and reach an agreement about what everyone should do, balancing their mutual interest in maximizing the total benefit and their conflicting interests in the division of gains. Such negotiations can take several rounds in which agreements are made on a tentative basis, better alternatives are explored, and the deal is finalized only when no group of players can find anything better. However, even after the completion of such a process, additional difficulties often arise in putting the final agreement into practice. For instance, all the players must perform, in the end, the actions that were stipulated for them in the agreement. When all others do what they are supposed to do, any one participant can typically get a better outcome for himself by doing something different. And, if each one suspects that the others may cheat in this way, he would be foolish to adhere to his stipulated cooperative action.

Agreements to cooperate can succeed if all players act immediately and in the presence of the whole group, but agreements with such immediate implementation are quite rare. More often the participants disperse after the agreement has been reached and then take their actions in private. Still, if these actions are observable to the others, and a third party—for example, a court of law—can enforce compliance, then the agreement of joint action can prevail.

However, in many other instances individual actions are neither directly observable nor enforceable by external forces. Without enforceability, agreements will stand only if it is in all participants' individual interests to abide by them. Games among sovereign countries are of this kind, as are many games with private information or games where the actions are either outside the law or too trivial or too costly to enforce in a court of law. In fact, games where agreements for joint action are not enforceable constitute a vast majority of strategic interactions.

Game theory uses a special terminology to capture the distinction between situations in which agreements are enforceable and those in which they are not. Games in which joint-action agreements are enforceable are called **cooperative games**; those in which such enforcement is not possible, and individual participants must be allowed to act in their own interests, are called **noncooperative games**. This has become standard terminology, but it is somewhat unfortunate because it gives the impression that the former will produce cooperative outcomes and the latter will not. In fact, individual action can be compatible with the achievement of a lot of mutual gain, especially in repeated interactions. The important distinction is that in so-called noncooperative games, cooperation will emerge only if it is in the participants' separate and individual interests to continue to take the prescribed actions. This emergence of cooperative outcomes from noncooperative behavior is one of the most interesting findings of game theory, and we will develop the idea in Chapters 10, 11, and 12.

We will adhere to the standard usage, but emphasize that the terms *cooperative* and *noncooperative* refer to the way in which actions are implemented or enforced—collectively in the former mode and individually in the latter—and not to the nature of the outcomes.

As we said earlier, most games in practice do not have adequate mechanisms for external enforcement of joint-action agreements. Therefore, most of our analytical discussion will deal with the noncooperative mode. The one exception comes in our discussion of bargaining in Chapter 17.

3 SOME TERMINOLOGY AND BACKGROUND ASSUMPTIONS

When one thinks about a strategic game, the logical place to begin is by specifying its structure. This includes all the strategies available to all the players, their information, and their objectives. The first two aspects will differ from one game to another along the dimensions discussed in the preceding section, and one must locate one's particular game within that framework. The objectives raise some new and interesting considerations. Here we consider aspects of all these matters.

A. Strategies

Strategies are simply the choices available to the players, but even this basic notion requires some further study and elaboration. If a game has purely simultaneous moves made only once, then each player's strategy is just the action taken on that single occasion. But if a game has sequential moves, then a player who moves later in the game can respond to what other players have done (or what he himself has done) at earlier points. Therefore, each such player must make a complete plan of action, for example: "If the other does A, then I will do X, but if the other does B, then I will do Y." This complete plan of action constitutes the strategy in such a game.

A very simple test determines whether your strategy is complete: Does it specify such full detail about how you would play the game—describing your action in every contingency—that, if you were to write it all down, hand it to someone else, and go on vacation, this other person acting as your representative could play the game just as you would have played it? He would know what to do on each occasion that could conceivably arise in the course of play without ever needing to disturb your vacation for instructions on how to deal with some situation that you had not foreseen.

This test will become clearer in Chapter 3, when we develop and apply it in some specific contexts. For now, you should simply remember that a strategy is a complete plan of action.

This notion is similar to the common usage of the word *strategy* to denote a longer-term or larger-scale plan of action, as distinct from tactics that pertain to a shorter term or a smaller scale. For example, generals in the military make strategic plans for a war or a large-scale battle, while lower-level officers devise tactics for a smaller skirmish or a particular theater of battle based on local conditions. But game theory does not use the term *tactics* at all. The term *strategy* covers all the situations, meaning a complete plan of action when necessary and meaning a single move if that is all that is needed in the particular game being studied.

The word *strategy* is also commonly used to describe a person's decisions over a fairly long time span and sequence of choices, even though there is no game in our sense of purposive interaction with other people. Thus, you have probably already chosen a career strategy. When you start earning an income, you will make saving and investment strategies and eventually plan a retirement strategy. This usage of the term *strategy* has the same sense as ours—a plan for a succession of actions in response to evolving circumstances. The only difference is that we are reserving it for a situation—namely, a game—where the circumstances evolve because of actions taken by other purposive players.

B. Payoffs

When asked what a player's objective in a game is, most newcomers to strategic thinking respond that it is "to win," but matters are not always so simple. Sometimes the margin of victory matters; for example, in R&D competition, if your product is only slightly better than the nearest rival's, your patent may be more open to challenge. Sometimes there may be smaller prizes for several participants, so winning isn't everything. Most important, very few games of strategy are purely zero-sum or win-lose; they combine some common interest and some conflict among the players. Thinking about such mixed-motive games requires more refined calculations than the simple dichotomy of winning and losing—for example, comparisons of the gains from cooperating versus cheating.

We will give each player a complete numerical scale with which to compare all logically conceivable outcomes of the game, corresponding to each available combination of choices of strategies by all the players. The number associated with each possible outcome will be called that player's **payoff** for that outcome. Higher payoff numbers attach to outcomes that are better in this player's rating system.

Sometimes the payoffs will be simple numerical ratings of the outcomes, the worst labeled 1, the next worst 2, and so on, all the way to the best. In other games, there may be more natural numerical scales—for example, money income or profit for firms, viewer-share ratings for television networks, and so on. In many situations, the payoff numbers are only educated guesses. In such cases, we need to check that the results of our analysis do not change significantly if we vary these guesses within some reasonable margin of error.

Two important points about the payoffs need to be understood clearly. First, the payoffs for one player capture everything in the outcomes of the game that he cares about. In particular, the player need not be selfish, but his concern about others should be already included in his numerical payoff scale. Second, we will suppose that, if the player faces a random prospect of outcomes, then the number associated with this prospect is the average of the payoffs associated with each component outcome, each weighted by its probability. Thus, if in one player's ranking, outcome A has payoff 0 and outcome B has payoff 100, then the prospect of a 75% probability of A and a 25% probability of B should have the payoff $0.75 \times 0 + 0.25 \times 100 = 25$. This is often called the **expected payoff** from the random prospect. The word *expected* has a special connotation in the jargon of probability theory. It does not mean what you think you will get or expect to get; it is the mathematical or probabilistic or statistical expectation, meaning an average of all possible outcomes, where each is given a weight proportional to its probability.

The second point creates a potential difficulty. Consider a game where players get or lose money and payoffs are measured simply in money amounts. In reference to the preceding example, if a player has a 75% chance of getting nothing and a 25% chance of getting $100, then the expected payoff as calculated in that example is $25. That is also the payoff that the player would get from a simple nonrandom outcome of $25. In other words, in this way of calculating payoffs, a person should be indifferent to whether he receives $25 for sure or faces a risky prospect of which the average is $25. One would think that most people would be averse to risk, preferring a sure $25 to a gamble that yields only $25 on the average.

A very simple modification of our payoff calculation gets around this difficulty. We measure payoffs not in money sums but by using a nonlinear rescaling of the dollar amounts. This is called the expected utility approach, and we will present it in detail in the appendix to Chapter 7. For now, please take our word that incorporating differing attitudes toward risk into our framework is a manageable task. Almost all of game theory is based on the expected utility approach, and it is indeed very useful, although not without flaws. We will adopt it in this book, but we also will indicate some of the difficulties that it leaves unresolved, with the use of a simple example in Chapter 7, Section 5.C.

C. Rationality

Each player's aim in the game will be to achieve as high a payoff for himself as possible. But how good is each player at pursuing this aim? This question is not about whether and how other players pursuing their own interests will impede him; that is in the very nature of a game of strategic interaction. Rather, achieving a high payoff is based on how good each player is at calculating the strategy that is in his own best interests and at following this strategy in the actual course of play.

Much of game theory assumes that players are perfect calculators and flawless followers of their best strategies. This is the assumption of **rational behavior**. Observe the precise sense in which the term *rational* is being used. It means that each player has a consistent set of rankings (values or payoffs) over all the logically possible outcomes and calculates the strategy that best serves these interests. Thus rationality has two essential ingredients: complete knowledge of one's own interests, and flawless calculation of what actions will best serve those interests.

It is equally important to understand what is *not* included in this concept of rational behavior. It does not mean that players are selfish; a player may rate highly the well-being of some other player(s) and incorporate this high rating into his payoffs. It does not mean that players are short-term thinkers; in fact, calculation of future consequences is an important part of strategic thinking, and actions that seem irrational from an immediate perspective may have valuable long-term strategic roles. Most important, being rational does not mean having the same value system as other players, or sensible people, or ethical or moral people would use; it means merely pursuing one's own value system consistently. Therefore, when one player carries out an analysis of how other players will respond (in a game with sequential moves) or of the successive rounds of thinking about thinking (in a game with simultaneous moves), he must recognize that the other players calculate the consequences of their choices by using their own value or rating system. You must not impute your own value systems or standards of rationality to others and assume that they would act as you would in that situation. Thus, many "experts" commenting on the Persian Gulf conflict in late 1990 and again in 2002–2003 predicted that Saddam Hussein would back down "because he is rational"; they failed to recognize that Saddam's value system was different from the one held by most Western governments and by the Western experts.

In general, each player does not really know the other players' value systems; this is part of the reason that in reality many games have incomplete and asymmetric information. In such games, trying to find out the values of others and trying to conceal or convey one's own become important components of strategy.

Game theory assumes that all players are rational. How good is this assumption, and therefore how good is the theory that employs it? At one level, it is obvious that the assumption cannot be literally true. People often don't even have full advance knowledge of their own value systems; they don't think ahead about how they would rank hypothetical alternatives and then remember these rankings until they are actually confronted with a concrete choice. Therefore they find it very difficult to perform the logical feat of tracing all possible consequences of their and other players' conceivable strategic choices and ranking the outcomes in advance in order to choose which strategy to follow. Even if they knew their preferences, the calculation would remain far from easy. Most games in real life are very complex, and most real players are limited in their thinking and computational abilities. In games such as chess, it is known that the calculation for the best strategy can be performed in a finite number of steps, but that number is so large that no one has succeeded in performing it, and good play remains largely an art.

The assumption of rationality may be closer to reality when the players are regulars who play the game quite often. Then they benefit from having experienced the different possible outcomes. They understand how the strategic choices of various players lead to the outcomes and how well or badly they themselves fare. Then we can hope that their choices, even if not made with full and conscious computations, closely approximate the results of such computations. We can think of the players as implicitly choosing the optimal strategy or behaving as if they were perfect calculators. We will offer some experimental evidence in Chapter 5 that the experience of playing the game generates more rational behavior.

The advantage of making a complete calculation of your best strategy, taking into account the corresponding calculations of a similar strategically calculating rival, is that then you are not making mistakes that the rival can exploit. In many actual situations, you may have specific knowledge of the way in which the other players fall short of this standard of rationality, and you can exploit this in devising your own strategy. We will say something about such calculations, but very often this is a part of the “art” of game playing, not easily codifiable in rules to be followed. You must always beware of the danger that the others are merely pretending to have poor skills or strategy, losing small sums through bad play and hoping that you will then raise the stakes, when they can raise the level of their play and exploit your gullibility. When this risk is real, the safer advice to a player may be to assume that the rivals are perfect and rational calculators and to choose his own best response to them. In other words, you should play to your opponents’ capabilities instead of their limitations.

D. Common Knowledge of Rules

We suppose that, at some level, the players have a common understanding of the rules of the game. In a *Peanuts* cartoon, Lucy thought that body checking was allowed in golf and decked Charlie Brown just as he was about to take his swing. We do not allow this.

The qualification “at some level” is important. We saw how the rules of the immediate game could be manipulated. But this merely admits that there is another game being played at a deeper level—namely, where the players choose the rules of the superficial game. Then the question is whether the rules of this deeper game are fixed. For example, in the legislative context, what are the rules of the agenda-setting game? They may be that the committee chairs have the power. Then how are the committees and their chairs elected? And so on. At some basic level, the rules are fixed by the constitution, by the technology of campaigning, or by general social norms of behavior. We ask that all players recognize the given rules of this basic game, and that is the focus of the analysis. Of course, that is an ideal; in practice, we may not be able to proceed to a deep enough level of analysis.

Strictly speaking, the rules of the game consist of (1) the list of players, (2) the strategies available to each player, (3) the payoffs of each player for all possible combinations of strategies pursued by all the players, and (4) the assumption that each player is a rational maximizer.

Game theory cannot properly analyze a situation where one player does not know whether another player is participating in the game, what the entire sets of actions available to the other players are from which they can choose, what their value systems are, or whether they are conscious maximizers of their own payoffs. But in actual strategic interactions, some of the biggest gains are to be made by taking advantage of the element of surprise and doing something that your rivals never thought you capable of. Several vivid examples can be found in historic military conflicts. For example, in 1967 Israel launched a preemptive attack that destroyed the Egyptian air force on the ground; in 1973 it was Egypt's turn to spring a surprise by launching a tank attack across the Suez Canal.

It would seem, then, that the strict definition of game theory leaves out a very important aspect of strategic behavior, but in fact matters are not that bad. The theory can be reformulated so that each player attaches some small probability to the situation where such dramatically different strategies are available to the other players. Of course, each player knows his own set of available strategies. Therefore, the game becomes one of asymmetric information and can be handled by using the methods developed in Chapter 8.

The concept of common knowledge itself requires some explanation. For some fact or situation X to be common knowledge between two people, A and B, it is not enough for each of them separately to know X. Each should also know that the other knows X; otherwise, for example, A might think that B does not know X and might act under this misapprehension in the midst of a game. But then A should also know that B knows that A knows X, and the other way around, otherwise A might mistakenly try to exploit B's supposed ignorance of A's knowledge. Of course, it doesn't even stop there. A should know that B knows that A knows that B knows, and so on ad infinitum. Philosophers have a lot of fun exploring the fine points of this infinite regress and the intellectual paradoxes that it can generate. For us, the general notion that the players have a common understanding of the rules of their game will suffice.

E. Equilibrium

Finally, what happens when rational players' strategies interact? Our answer will generally be in the framework of **equilibrium**. This simply means that each player is using the strategy that is the best response to the strategies of the other players. We will develop game-theoretic concepts of equilibrium in Chapters 3 through 7 and then use them in subsequent chapters.

Equilibrium does not mean that things don't change; in sequential-move games the players' strategies are the complete plans of action and reaction, and the position evolves all the time as the successive moves are made and responded to. Nor does equilibrium mean that everything is for the best; the interaction of rational strategic choices by all players can lead to bad outcomes for all, as in the prisoners' dilemma. But we will generally find that the idea of equilibrium is a useful descriptive tool and organizing concept for our analysis. We will consider this idea in greater detail later, in connection with specific equilibrium concepts. We will also see how the concept of equilibrium can be augmented or modified to remove some of its flaws and to incorporate behavior that falls short of full calculating rationality.

Just as the rational behavior of individual players can be the result of experience in playing the game, fitting of their choices into an overall equilibrium can come about after some plays that involve trial and error and nonequilibrium outcomes. We will look at this matter in Chapter 5.

Defining an equilibrium is not hard, but finding an equilibrium in a particular game—that is, solving the game—can be a lot harder. Throughout this book, we will solve many simple games in which there are two or three players, each of them having two or three strategies or one move each in turn. Many people believe this to be the limit of the reach of game theory and therefore believe that the theory is useless for the more complex games that take place in reality. That is not true.

Humans are severely limited in their speed of calculation and in their patience for performing long calculations. Therefore, humans can easily solve only the simple games with two or three players and strategies. But computers are very good at speedy and lengthy calculations. Many games that are far beyond the power of human calculators are easy for computers. The level of complexity in many games in business and politics is already within the power of computers. Even in games such as chess that are far too complex to solve completely, computers have reached a level of ability comparable to that of the best humans; we consider chess in more detail in Chapter 3.

Computer programs for solving quite complex games exist, and more are appearing rapidly. Mathematica and similar program packages contain routines for finding mixed-strategy equilibria in simultaneous-move games. Gambit, a National Science Foundation project led by Professors Richard D. McKelvey of the California Institute of Technology and Andrew McLennan of the University of Minnesota, is producing a comprehensive set of routines for finding equilibria in sequential- and simultaneous-move games, in pure and mixed strategies, and with varying degrees of uncertainty and incomplete information. We will refer to this project again in several places in the next several chapters. The biggest advantage of the project is that its programs are open source and can easily be obtained from its Web site www.gambit-project.org.

Why then do we set up and solve several simple games in detail in this book? The reason is that understanding the concepts is an important prerequisite for making good use of the mechanical solutions that computers can deliver, and understanding comes from doing simple cases yourself. This is exactly how you learned and now use arithmetic. You came to understand the ideas of addition, subtraction, multiplication, and division by doing many simple problems mentally or using paper and pencil. With this grasp of basic concepts, you can now use calculators and computers to do far more complicated sums than you would ever have the time or patience to do manually. But if you did not understand the concepts, you would make errors in using calculators; for example, you might solve $3 + 4 \times 5$ by grouping additions and multiplications incorrectly as $(3 + 4) \times 5 = 35$ instead of correctly as $3 + (4 \times 5) = 23$.

Thus, the first step of understanding the concepts and tools is essential. Without it, you would never learn to set up correctly the games that you ask the computer to solve. You would not be able to inspect the solution with any feeling for whether it was reasonable, and if it was not, you would not be able to go back to your original specification, improve it, and solve it again until the specification and the calculation correctly captured the strategic situation that you wanted to study. Therefore, please pay serious attention to the simple examples that we solve and the drill exercises that we ask you to solve, especially in Chapters 3 through 7.

F. Dynamics and Evolutionary Games

The theory of games based on assumptions of rationality and equilibrium has proved very useful, but it would be a mistake to rely on it totally. When games are played by novices who do not have the necessary experience to perform the calculations to choose their best strategies, explicitly or implicitly, their choices, and therefore the outcome of the game, can differ significantly from the predictions of analysis based on the concept of equilibrium.

However, we should not abandon all notions of good choice; we should recognize the fact that even poor calculators are motivated to do better for their own sakes and will learn from experience and by observing others. We should allow for a dynamic process in which strategies that proved to be better in previous plays of the game are more likely to be chosen in later plays.

The **evolutionary** approach to games does just this. It is derived from the idea of evolution in biology. Any individual animal's genes strongly influence its behavior. Some behaviors succeed better in the prevailing environment, in the sense that the animals exhibiting those behaviors are more likely to reproduce successfully and pass their genes to their progeny. An evolutionary stable state, relative to a given environment, is the ultimate outcome of this process over several generations.

The analogy in games would be to suppose that strategies are not chosen by conscious rational maximizers, but instead that each player comes to the game with a particular strategy "hardwired" or "programmed" in. The players then confront other players who may be programmed to apply the same or different strategies. The payoffs to all the players in such games are then obtained. The strategies that fare better—in the sense that the players programmed to play them get higher payoffs in the games—multiply faster, whereas the strategies that fare worse decline. In biology, the mechanism of this growth or decay is purely genetic transmission through reproduction. In the context of strategic games in business and society, the mechanism is much more likely to be social or cultural—observation and imitation, teaching and learning, greater availability of capital for the more successful ventures, and so on.

The object of study is the dynamics of this process. Does it converge to an evolutionary stable state? Does just one strategy prevail over all others in the end, or can a few strategies coexist? Interestingly, in many games, the evolutionary stable limit is the same as the equilibrium that would result if the players were consciously rational calculators. Therefore, the evolutionary approach gives us a backdoor justification for equilibrium analysis.

The concept of evolutionary games has thus imported biological ideas into game theory; there has been an influence in the opposite direction, too. Biologists have recognized that significant parts of animal behavior consist of strategic interactions with other animals. Members of a given species compete with one another for space or mates; members of different species relate to one another as predators and prey along a food chain. The payoff in such games in turn contributes to reproductive success and therefore to biological evolution. Just as game theory has benefited by importing ideas from biological evolution for its analysis of choice and dynamics, biology has benefited by importing game-theoretic ideas of strategies and payoffs for its characterization of basic interactions between animals. We have a true instance of synergy or symbiosis. We provide an introduction to the study of evolutionary games in Chapter 12.

G. Observation and Experiment

All of Section 3 to this point has concerned how to think about games or how to analyze strategic interactions. This constitutes theory. This book will cover an extremely simple level of theory, developed through cases and illustrations instead of formal mathematics or theorems, but it will be theory just the same. All theory should relate to reality in two ways. Reality should help structure the theory, and reality should provide a check on the results of the theory.

We can find out the reality of strategic interactions in two ways: (1) by observing them as they occur naturally and (2) by conducting special experiments that help us pin down the effects of particular conditions. Both methods have been used, and we will mention several examples of each in the proper contexts.

Many people have studied strategic interactions—the participants' behavior and the outcomes—under experimental conditions, in classrooms among "captive" players, or in special laboratories with volunteers. Auctions, bargaining, prisoners' dilemmas, and several other games have been studied in this way. The results are a mixture. Some conclusions of the theoretical analysis are borne out; for example, in games of buying and selling, the participants generally settle quickly on the economic equilibrium. In other contexts, the outcomes differ significantly from the theoretical predictions; for example, prisoners' dilemmas and bargaining games show more cooperation than theory based on the assumption of selfish, maximizing behavior would lead us to expect, whereas auctions show some gross overbidding.

At several points in the chapters that follow, we will review the knowledge that has been gained by observation and experiments, discuss how it relates to the theory, and consider what reinterpretations, extensions, and modifications of the theory have been made or should be made in light of this knowledge.

4 THE USES OF GAME THEORY

We began Chapter 1 by saying that games of strategy are everywhere—in your personal and working life; in the functioning of the economy, society, and polity around you; in sports and other serious pursuits; in war and in peace. This should be motivation enough to study such games systematically, and that is what game theory is about. But your study can be better directed if you have a clearer idea of just how you can put game theory to use. We suggest a threefold perspective.

The first use is in *explanation*. Many events and outcomes prompt us to ask: Why did that happen? When the situation requires the interaction of decision makers with different aims, game theory often supplies the key to understanding the situation. For example, cutthroat competition in business is the result of the rivals being trapped in a prisoners' dilemma. At several points in the book, we will mention actual cases where game theory helps us to understand how and why the events unfolded as they did. This includes Chapter 14's detailed case study of the Cuban missile crisis from the perspective of game theory.

The other two uses evolve naturally from the first. The second is in *prediction*. When looking ahead to situations where multiple decision makers will interact strategically, we can use game theory to foresee what actions they will take and what outcomes will result. Of course, prediction for a particular context depends on its details, but we will prepare you to use prediction by analyzing several broad classes of games that arise in many applications.

The third use is in *advice* or *prescription.* We can act in the service of one participant in the future interaction and tell him which strategies are likely to yield good results and which ones are likely to lead to disaster. Once again such work is context specific, and we can equip you with several general principles and techniques and show you how to apply them to some general types of contexts. For example, in Chapter 7, we will show how to mix moves; in Chapter 9, we will examine how to make your commitments, threats, and promises credible; in Chapter 10, we will examine alternative ways of overcoming prisoners' dilemmas.

The theory is far from perfect in performing any of the three functions. To explain an outcome, one must first have a correct understanding of the motives and behavior of the participants. As we saw earlier, most of game theory takes a specific approach to these matters—namely, the framework of rational choice of individual players and the equilibrium of their interaction. Actual players and interactions in a game might not conform to this framework. But the proof of the pudding is in the eating. Game-theoretic analysis has greatly improved our understanding of many phenomena, as reading this book should convince you. The theory continues to evolve and improve as the result of ongoing research. This book will equip you with the basics so that you can more easily learn and profit from the new advances as they appear.

When explaining a past event, we can often use historical records to get a good idea of the motives and the behavior of the players in the game. When attempting prediction or advice, there is the additional problem of determining what motives will drive the players' actions, what informational or other limitations they will face, and sometimes even who the players will be. Most important, if game-theoretic analysis assumes that the other player is a rational maximizer of his own objectives when in fact he is unable to do the calculations or is a clueless person acting at random, the advice based on that analysis may prove wrong. This risk is reduced as more and more players recognize the importance of strategic interaction and think through their strategic choices or get expert advice on the matter, but some risk remains. Even then, the systematic thinking made possible by the framework of game theory helps keep the errors down to this irreducible minimum, by eliminating the errors that arise from faulty logical thinking about the strategic interaction. Also, game theory can take into account many kinds of uncertainty and incomplete information, including that about the strategic possibilities and rationality of the opponent. We will consider a few examples in the chapters to come.

5 THE STRUCTURE OF THE CHAPTERS TO FOLLOW

In this chapter, we introduced several considerations that arise in almost every game in reality. To understand or predict the outcome of any game, we must know in greater detail all of these ideas. We also introduced some basic concepts that will prove useful in such analysis. However, trying to cope with all of the concepts at once merely leads to confusion and a failure to grasp any of them. Therefore, we will build up the theory one concept at a time. We will develop the appropriate technique for analyzing that concept and illustrate it.

In the first group of chapters, from Chapters 3 to 7, we will construct and illustrate the most important of these concepts and techniques. We will examine purely sequential-move games in Chapter 3 and introduce the techniques—game trees and rollback reasoning—that are used to analyze and solve such games. In Chapters 4 and 5, we will turn to games with simultaneous moves and develop for them another set of concepts—payoff tables, dominance, and Nash equilibrium. Both chapters will focus on games where players use pure strategies; in Chapter 4, we will restrict players to a finite set of pure strategies, and in Chapter 5, we will allow strategies that are continuous variables. Chapter 5 will also examine some mixed empirical evidence and conceptual criticisms and counterarguments on Nash equilibrium, and a prominent alternative to Nash equilibrium—namely, rationalizability. In Chapter 6, we will show how games that have some sequential moves and some simultaneous moves can be studied by combining the techniques developed in Chapters 3 through 5. In Chapter 7, we will turn to simultaneous-move games that require the use of randomization or mixed strategies. We will start by introducing the basic ideas about mixing in two-by-two games, develop the simplest techniques for finding mixed-strategy Nash equilibria, and then consider more complex examples along with the empirical evidence on mixing.

The ideas and techniques developed in Chapters 3 through 7 are the most basic ones: (1) correct forward-looking reasoning for sequential-move games, and (2) equilibrium strategies—pure and mixed—for simultaneous-move games. Equipped with these concepts and tools, we can apply them to study some broad classes of games and strategies in Chapters 8 through 12.

Chapter 8 studies what happens in games when players are subject to uncertainty or when they have asymmetric information. We will examine strategies for coping with risk and even for using risk strategically. We will also study the important strategies of signaling and screening that are used for manipulating and eliciting information. We will develop the appropriate generalization of Nash equilibrium in the context of uncertainty, namely Bayesian Nash equilibrium, and show the different kinds of equilibria that can arise. In Chapter 9, we will continue to examine the role of player manipulation in games as we consider strategies that players use to manipulate the rules of a game, by seizing a first-mover advantage and making a strategic move. Such moves are of three kinds—commitments, threats, and promises. In each case, credibility is essential to the success of the move, and we will outline some ways of making such moves credible.

In Chapter 10, we will move on to study the best-known game of them all—the prisoners' dilemma. We will study whether and how cooperation can be sustained, most importantly in a repeated or ongoing relationship. Then, in Chapter 11, we will turn to situations where large populations, rather than pairs or small groups of players, interact strategically, games that concern problems of collective action. Each person's actions have an effect—in some instances beneficial, in others, harmful—on the others. The outcomes are generally not the best from the aggregate perspective of the society as a whole. We will clarify the nature of these outcomes and describe some simple policies that can lead to better outcomes.

All these theories and applications are based on the supposition that the players in a game fully understand the nature of the game and deploy calculated strategies that best serve their objectives in the game. Such rationally optimal behavior is sometimes too demanding of information and calculating power to be believable as a good description of how people really act. Therefore, Chapter 12 will look at games from a very different perspective. Here, the players are not calculating and do not pursue optimal strategies. Instead, each player is tied, as if genetically preordained, to a particular strategy. The population is diverse, and different players have different predetermined strategies. When players from such a population meet and act out their strategies, which strategies perform better? And if the more successful strategies proliferate better in the population, whether through inheritance or imitation, then what will the eventual structure of the population look like? It turns out that such evolutionary dynamics often favor exactly those strategies that would be used by rational optimizing players. Thus, our study of evolutionary games lends useful indirect support to the theories of optimal strategic choice and equilibrium that we will have studied in the previous chapters.

In the final group, Chapters 13 through 17, we will take up specific applications to situations of strategic interactions. Here, we will use as needed the ideas and methods from all the earlier chapters. Chapter 13 uses the methods developed in Chapter 8 to analyze the strategies that people and firms have to use when dealing with others who have some private information. We will illustrate the screening mechanisms that are used for eliciting information—for example, the multiple fares with different restrictions that airlines use for separating the business travelers who are willing to pay more from the tourists who are more price sensitive. We will also develop the methods for designing incentive payments to elicit effort from workers when direct monitoring is difficult or too costly. Chapter 14 then applies the ideas from Chapter 9 to examine a particularly interesting dynamic version of a threat, known as the strategy of brinkmanship. We will elucidate its nature and apply the idea to study the Cuban missile crisis of 1962. Chapter 15 is about voting in committees and elections. We will look at the variety of voting rules available and some paradoxical results that can arise. In addition, we will address the potential for strategic behavior not only by voters but also by candidates in a variety of election types.

Chapters 16 and 17 will look at mechanisms for the allocation of valuable economic resources: Chapter 16 will treat auctions and Chapter 17 will consider bargaining processes. In our discussion of auctions, we will emphasize the roles of information and attitudes toward risk in the formulation of optimal strategies for both bidders and sellers. We will also take the opportunity to apply the theory to the newest type of auctions, those that take place online. Finally, Chapter 17 will present bargaining in both cooperative and noncooperative settings.

All of these chapters together provide a lot of material; how might readers or teachers with more specialized interests choose from it? Chapters 3 through 7 constitute the core theoretical ideas that are needed throughout the rest of the book. Chapters 9 and 10 are likewise important for the general classes of games and strategies considered therein. Beyond that, there is a lot from which to pick and choose. Section 1 of Chapter 5, Section 7 of Chapter 7, Section 5 of Chapter 10, and Section 7 of Chapter 12, for example, all consider more advanced topics. These sections will appeal to those with more scientific and quantitative backgrounds and interests, but those who come from the social sciences or humanities and have less quantitative background can omit them without loss of continuity. Chapter 8 deals with an important topic in that most games in practice have incomplete and asymmetric information, and the players' attempts to manipulate information is a critical aspect of many strategic interactions. However, the concepts and techniques for analyzing information games are inherently somewhat more complex. Therefore, some readers and teachers may choose to study just the examples that convey the basic ideas of signaling and screening and leave out the rest. We have placed this chapter early in Part Three, however, in view of the importance of the subject. Chapters 9 and 10 are key to understanding many phenomena in the real world, and most teachers will want to include them in their courses, but Section 5 of Chapter 10 is mathematically a little more advanced and can be omitted. Chapters 11 and 12 both look at games with large numbers of players. In Chapter 11, the focus is on social interactions; in Chapter 12, the focus is on evolutionary biology. The topics in Chapter 12 will be of greatest interest to those with interests in biology, but similar themes are emerging in the social sciences, and students from that background should aim to get the gist of the ideas even if they skip the details. Chapter 13 is most important for students of business and organization theories. Chapters 14 and 15 present topics from political science (international diplomacy and elections, respectively), and Chapters 16 and 17 cover topics from economics (auctions and bargaining). Those teaching courses with more specialized audiences may choose a subset from Chapters 11 through 17, and indeed expand on the ideas considered therein.

Whether you come from mathematics, biology, economics, politics, other sciences, or from history or sociology, the theory and examples of strategic games will stimulate and challenge your intellect. We urge you to enjoy the subject even as you are studying or teaching it.

SUMMARY

Strategic *games* situations are distinguished from individual decision-making situations by the presence of significant interactions among the players. Games can be classified according to a variety of categories including the timing of play, the common or conflicting interests of players, the number of times an interaction occurs, the amount of information available to the players, the type of rules, and the feasibility of coordinated action.

Learning the terminology for a game's structure is crucial for analysis. Players have *strategies* that lead to different *outcomes* with different associated *payoffs*. Payoffs incorporate everything that is important to a player about a game and are calculated by using probabilistic averages or *expectations* if outcomes are random or include some risk. *Rationality*, or consistent behavior, is assumed of all players, who must also be aware of all of the relevant rules of conduct. *Equilibrium* arises when all players use strategies that are best responses to others' strategies; some classes of games allow learning from experience and the study of dynamic movements toward equilibrium. The study of behavior in actual game situations provides additional information about the performance of the theory.

Game theory may be used for explanation, prediction, or prescription in various circumstances. Although not perfect in any of these roles, the theory continues to evolve; the importance of strategic interaction and strategic thinking has also become more widely understood and accepted.

KEY TERMS[3]

asymmetric information (23)
constant-sum game (21)
cooperative game (26)
decision (18)
equilibrium (32)
evolutionary game (34)
expected payoff (29)
external uncertainty (23)

[3] The number in parentheses after each key term is the page on which that term is defined or discussed.

game (18)
imperfect information (23)
noncooperative game (26)
payoff (28)
perfect information (23)
rational behavior (30)
screening (24)
screening device (24)
sequential moves (20)
signal (24)
signaling (24)
simultaneous moves (20)
strategic game (18)
strategic uncertainty (23)
strategy (27)
zero-sum game (21)

SOLVED EXERCISES[4]

S1. Determine which of the following situations describe games and which describe decisions. In each case, indicate what specific features of the situation caused you to classify it as you did.

(a) A group of grocery shoppers in the dairy section, with each shopper choosing a flavor of yogurt to purchase

(b) A pair of teenage girls choosing dresses for their prom

(c) A college student considering what type of postgraduate education to pursue

(d) The *New York Times* and the *Wall Street Journal* choosing the prices for their online subscriptions this year

(e) A presidential candidate picking a running mate

S2. Consider the strategic games described below. In each case, state how you would classify the game according to the six dimensions outlined in the text. (i) Are moves sequential or simultaneous? (ii) Is the game zero-sum or not? (iii) Is the game repeated? (iv) Is there imperfect information, and if so, is there incomplete (asymmetric) information? (v) Are the rules fixed or not? (vi) Are cooperative agreements possible or not? If you do not have enough information to classify a game in a particular dimension, explain why not.

(a) *Rock-Paper-Scissors*: On the count of three, each player makes the shape of one of the three items with his hand. Rock beats Scissors, Scissors beats Paper, and Paper beats Rock.

(b) *Roll-call voting*: Voters cast their votes orally as their names are called. The choice with the most votes wins.

(c) *Sealed-bid auction*: Bidders on a bottle of wine seal their bids in envelopes. The highest bidder wins the item and pays the amount of his bid.

[4] **Note to Students:** The solutions to the **Solved Exercises** are found on the Web site **wwnorton.com/books/games_of_strategy**, which is free and open to all.

S3. "A game player would never prefer an outcome in which every player gets a little profit to an outcome in which he gets all the available profit." Is this statement true or false? Explain why in one or two sentences.

S4. You and a rival are engaged in a game in which there are three possible outcomes: you win, your rival wins (you lose), or the two of you tie. You get a payoff of 50 if you win, a payoff of 20 if you tie, and a payoff of 0 if you lose. What is your expected payoff in each of the following situations?

(a) There is a 50% chance that the game ends in a tie, but only a 10% chance that you win. (There is thus a 40% chance that you lose.)

(b) There is a 50–50 chance that you win or lose. There are no ties.

(c) There is an 80% chance that you lose, a 10% chance that you win, and a 10% chance that you tie.

S5. Explain the difference between game theory's use as a predictive tool and its use as a prescriptive tool. In what types of real-world settings might these two uses be most important?

UNSOLVED EXERCISES

U1. Determine which of the following situations describe games and which describe decisions. In each case, indicate what specific features of the situation caused you to classify it as you did.

(a) A party nominee for president of the United States must choose whether to use private financing or public financing for her campaign.

(b) Frugal Fred receives a $20 gift card for downloadable music and must choose whether to purchase individual songs or whole albums.

(c) Beautiful Belle receives 100 replies to her online dating profile and must choose whether to reply to each of them.

(d) NBC chooses how to distribute its television shows online this season. The executives consider Amazon.com, iTunes, and/or NBC.com. The fee they might pay to Amazon or to iTunes is open to negotiation.

(e) China chooses a level of tariffs to apply to American imports.

U2. Consider the strategic games described below. In each case, state how you would classify the game according to the six dimensions outlined in the text. (i) Are moves sequential or simultaneous? (ii) Is the game zero-sum or not? (iii) Is the game repeated? (iv) Is there imperfect information, and if so, is there incomplete (asymmetric) information? (v) Are the rules fixed or not? (vi) Are cooperative agreements possible or not? If you do not have enough information to classify a game in a particular dimension, explain why not.

(a) Garry and Ross are sales representatives for the same company. Their manager informs them that of the two of them, whoever sells more this year wins a Cadillac.

(b) On the game show *The Price Is Right*, four contestants are asked to guess the price of a television set. Play starts with the leftmost player, and each player's guess must be different from the guesses of the previous players. The person who comes closest to the real price, without going over it, wins the television set.

(c) Six thousand players each pay $10,000 to enter the World Series of Poker. Each starts the tournament with $10,000 in chips, and they play No-Limit Texas Hold 'Em (a type of poker) until someone wins all the chips. The top 600 players each receive prize money according to the order of finish, with the winner receiving more than $8,000,000.

(d) Passengers on Desert Airlines are not assigned seats; passengers choose seats once they board. The airline assigns the order of boarding according to the time the passenger checks in, either on the Web site up to 24 hours before takeoff or in person at the airport.

U3. "Any gain by the winner must harm the loser." Is this statement true or false? Explain your reasoning in one or two sentences.

U4. Alice, Bob, and Confucius are bored during recess, so they decide to play a new game. Each of them puts a dollar in the pot, and each tosses a quarter. Alice wins if the coins land all heads or all tails. Bob wins if two heads and one tail land, and Confucius wins if one head and two tails land. The quarters are fair, and the winner receives a net payment of $2 ($3 − $1 = $2), and the losers lose their $1.

(a) What is the probability that Alice will win and the probability that she will lose?

(b) What is Alice's expected payoff?

(c) What is the probability that Confucius will win and the probability that he will lose?

(d) What is Confucius' expected payoff?

(e) Is this a zero-sum game? Please explain your answer.

U5. "When one player surprises another, this indicates that the players did not have common knowledge of the rules." Give an example that illustrates this statement, and give a counterexample that shows that the statement is not always true.