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Bidding Strategy and Auction Design

AUCTIONS AS MECHANISMS for selling goods and services date back to ancient Greece and Rome, where slaves and wives were commonly bought and sold at well-known public auction sites. Although the auction waned as a sales mechanism for several centuries after the fall of the Roman Empire, it regained popularity in eighteenth-century Britain and has been a common, if not ubiquitous, method of commerce since that time. Many thousands of people now make purchases at online auctions every day, and some may buy other items by way of mechanisms that are not even recognized as auctions.

Despite this long history, the first formal analysis of auctions dates only to 1961 and the path-breaking work of Nobel Prize winner William Vickrey. In the decades that followed, economists have devoted considerable energy to developing a better understanding of sales by auction, from the standpoint of both buyers (bidding strategy) and sellers (auction design). We cover both topics and provide a primer on auction rules and environments in this chapter.

Technically, the term "auction" refers to any transaction where the final price of the object for sale is arrived at by way of competitive bidding. Many different types of transactions fit this description. For example, the historic Filene's Basement department store in Boston used a clever pricing strategy to keep customers coming back for more: it reduced the prices on items remaining on the racks successively each week until either the goods were purchased or the price got so low that it donated the items to charity. Shoppers loved it. Little did they realize that they were participating in what is known as a descending, or Dutch, auction—one of the types of auctions described in detail in this chapter.

Even if you do not personally participate in many auctions, your life is greatly influenced by them. Since 1994, the Federal Communications Commission (FCC) has auctioned off large parts of the electromagnetic broadcasting spectrum in more than 75 different auctions. These auctions have raised approximately $80 billion in government revenues. Because these revenues have made significant contributions to the federal budget, they have affected important macroeconomic magnitudes, such as interest rates. International variables also have been affected not only by the U.S. spectrum auctions, but also by similar auctions in at least six European countries and in Australia and New Zealand. Understanding how auctions work will help you understand these important events and their implications.

From a strategic perspective, auctions have several characteristics of interest. Most crucial is the existence of asymmetric information between seller and bidders, as well as among bidders. Thus, signaling and screening can be important components of strategy for both bidders and sellers. In addition, optimal strategies for both bidders and sellers will depend on their levels of aversion to risk. We will also see that under some specific circumstances, expected payoffs to the seller as well as to the winning bidder are the same across auction types. The formal theory of auctions relies on advanced calculus to derive its results, but we eschew most of this difficult mathematics in favor of more intuitive descriptions of optimal behavior and strategy choice.[1]

1 TYPES OF AUCTIONS

Auctions differ in the methods used for the submission of bids and for the determination of the final price paid by the winner. These aspects of an auction, which are set in advance by the seller, are known as auction *rules*. In addition, auctions can be classified according to the type of object being auctioned and how it is valued; this determines the auction *environment*. Here we categorize the various auction rules and environments, describing their characteristics and mechanics.

A. Auction Rules

The seller generally determines the rules that will govern the auction. She has to do this with only limited knowledge of the bidders' willingness to pay. Therefore, the seller is in much the same position as the firm that tried to practice price discrimination or the government procurement officer that tried to find out the contractors' cost in Chapter 13. In other words, in choosing the rules, the seller

[1] A reference list of sources for additional information on the theory and practice of auctions can be found in the final section of the chapter.

is designing the mechanism of the auction. This mechanism-design approach can be developed into a theory of optimal auctions and also tell us when two or more such mechanisms will be equivalent. We must leave the general theory for more advanced texts; here we will study and compare a few specific mechanisms that are most frequently and prominently used in reality.[2]

The four major categories of auction rules can be divided into two groups. The first group is known as **open outcry**. Under this type of auction rule, bidders call out or otherwise make their bids in public. All bidders are able to observe bids as they are made. This type perhaps best fits the popular vision of the way in which auctions work—an image that includes feverish bidders and an auctioneer. But open outcry auctions can be organized in two ways. Only one of them would ever demonstrate "feverish" bidding.

The **ascending**, or **English**, version of an open-outcry auction conforms best to this popular impression of auctions. Ascending auctions are the norm at English auction houses such as Christie's and Sotheby's, from which they take their alternate name. The auction houses have a conventional auctioneer who starts at a low price and calls out successively higher prices for an item, waiting to receive a bid at each price before going on. When no further bids can be obtained, the item goes to the highest bidder. Thus, any number of bidders can take part in English auctions, although only the top bidder gains the item up for sale. And the bidding process may not literally entail the actual outcry of bids, because the mere nod of a head or the flick of a wrist is common bidding behavior in such auctions. Most Internet auction sites now run what are essentially ascending auctions in virtual, rather than real, time.

The other type of open outcry auction is the **Dutch**, or **descending**, auction. Dutch auctions, which get their name from the way in which tulips and other flowers are auctioned in the Netherlands, work in the opposite direction from that of English auctions. The auctioneer starts at an extremely high price and calls out successively lower prices until one of the assembled potential bidders accepts the price, makes a bid, and takes the item. Because of the desire or need for speed, Dutch flower auctions, as well as auctions for other agricultural or perishable goods (such as the daily auction at the Sydney Fish Market), use a "clock" that ticks down (counterclockwise) to ever lower prices until one bidder "stops the clock" and collects her merchandise. In many cases, the auction clock displays considerable information about the lot of goods currently for sale in addition to the falling price of those goods. And unlike in the English auction,

[2] Roger Myerson's paper "Optimal Auction Design," *Mathematics of Operations Research*, vol. 6, no. 1 (February 1981), pp. 58–73, was a pioneering contribution to the general theory of auctions and an important part of the work that won him the Nobel Prize in economics in 2007. Paul Klemperer, *Auctions: Theory and Practice* (Princeton: Princeton University Press, 2004), is an excellent modern treatment of the theory.

there is no feverish bidding in a Dutch auction, because only the one person who "stops the clock" takes any action.

The second group of auction rules requires sales to occur by **sealed bid**. In these auctions, bidding is done privately and bidders cannot observe any of the bids made by others; in many cases, only the winning bid is announced. Bidders in such auctions, as in Dutch auctions, have only one opportunity to bid. (Technically, you could submit multiple bids, but only the highest one would be relevant to the auction outcome.) Sealed-bid auctions have no need for an auctioneer. They require only an overseer who opens the bids and determines the winner.

Within sealed-bid auctions, there are two rules for determining the price paid by the high bidder. In a **first-price**, sealed-bid auction, the highest bidder wins the item and pays a price equal to her bid. In a **second-price**, sealed-bid auction, the highest bidder wins the item but pays a price equal to the bid of the second-highest bidder. A second-price rule can be extremely useful for eliciting truthful bids, as we will see in Section 4. Such an auction is often termed a **Vickrey auction** after the Nobel-prize-winning economist who first noted this particular characteristic. We will also see that the sealed-bid auctions are each similar, in regard to bidding strategy and expected payoffs, to one of the open-outcry auctions; first-price, sealed-bid auctions are similar to Dutch auctions, and second-price, sealed-bid auctions are similar to English auctions.

Other, less common, configurations also can be used to sell goods at auction. For example, you could set up an auction in which the highest bidder wins but the top two bidders pay their bids or one in which the high bidder wins but all bidders pay their bids, a procedure discussed in Section 5. We do not attempt to consider all possible combinations here. Rather, we analyze several of the most common auction schemes by using examples that bring out important strategic concepts.

B. Auction Environments

Finally, there are a number of ways in which bidders may value an item up for auction. The main distinction in such auction environments is based on the difference between *common-* and *private-value* objects. In a **common-value**, or **objective-value**, auction, the value of the object is the same for all the bidders, but each bidder generally knows only an imprecise estimate of it. Bidders may have some sense of the distribution of possible values, but each must form her own estimate before bidding. For example, an oil-drilling tract has a given amount of oil that should produce the same revenue for all companies, but each company has only its own expert's estimate of the amount of oil contained under the tract. Similarly, each bond trader has only an estimate of the future course of interest rates. In such auctions, signaling and screening can play an important role.

In a common-value auction, each bidder should be aware of the fact that other bidders possess some (however sketchy) information about an object's value, and she should attempt to infer the contents of that information from the actions of rival bidders. In addition, she should be aware of how her own actions might signal her private information to those rival bidders. When bidders' estimates of an object's value are influenced by their beliefs about other bidders' estimates, we have an environment in which bids are said to be *correlated* with each other. This situation has implications for both buyers and sellers as we will see later in this chapter.

In a **private-value**, or **subjective-value**, auction, bidders each determine their own individual value for an object. In this case, bidders place different values on an object. For example, a gown worn by Princess Diana or a necklace worn by Jacqueline Bouvier Kennedy Onassis may have sentimental value to some bidders. Bidders know their own private valuations in such auction environments but do not know each other's valuations of an object. Similarly, the seller does not know any of the bidders' valuations. Bidders and sellers may each be able to formulate rough estimates of others' valuations and, as above, can use signals and screens to attempt to improve their final outcomes. The information problem is relevant, then, not only to bidding strategies, but also to the seller's strategy in designing the form of auction to identify the highest valuation and to extract the best price.

2 THE WINNER'S CURSE

A standard but often ignored outcome arises in common-value auctions. Recall that such auctions entail the sale of an object whose value is fixed and identical for all bidders, although each bidder can only estimate it. The **winner's curse** is a warning to bidders that if they win the object in the auction, they are likely to have paid more than it is worth.

Suppose you are a corporate raider bidding for Targetco. Your experts have studied this company and produced estimates that, in the hands of the current management, it is worth somewhere between 0 and $10 billion, all values in this range being equally likely. The current management knows the precise figure, but of course it is not telling you. You believe that whatever Targetco is worth under existing management, it will be worth 50% more under your control. What should you bid?

You might be inclined to think that, on average, Targetco is worth $5 billion under existing management and thus $7.5 billion, on average, under yours. If so, then a bid somewhere between $5 billion and $7.5 billion should be profitable. But such a bidding strategy reckons without the response of the existing management to your bid. If Targetco is actually worth more than your bid, the current owners are not going to accept the bid. You are going to get the company only if its true worth is toward the lower end of the range.

Suppose you bid amount b. Your bid will be accepted and you will take over the management of Targetco if it is worth somewhere between 0 and b under the current management; on average, you can expect the company to be currently worth $b/2$ if your bid is accepted. In your hands, the average worth will be 50% more than the current worth, or $(1.5)(b/2) = 0.75b$. Because this value is always less than b, you would win the takeover battle only when it was not worth winning! Many raiders seem to have discovered this fact too late.

This result is not unlike that faced by the purchaser of a used car, which we discussed in Chapter 8. The theory of adverse selection in markets with asymmetric information is directly applicable to the common-value auction described here. Just as the average value of a used car will always fall below the price attached to the "good" cars, so will the average worth of Targetco in your hands always fall below your bid.

But corporate raiders, often engaged with target firms in one-on-one negotiations resembling auctions with only one bidder, are not the only ones affected by the winner's curse. Similar problems arise when you are competing with other bidders in a common-value auction and all of you have separate estimates for the object's value.

Consider a lease for the oil- or gas-drilling rights on a tract of land (or sea).[3] At the auction for this lease, you win only if your rivals make estimates of the value of the lease that are lower than your estimate. You should recognize this fact and try to learn from it.

Suppose the true value of the lease, unknown to any of the bidders, is \$1 billion. (In this case, the seller probably does not know the true value of the tract either.) Suppose there are 10 oil companies in the bidding. Each company's experts estimate the value of the tract with an error of \$100 million, all numbers in this range being equally likely. If all 10 of the estimates could be pooled, their arithmetic average would be an unbiased and much more accurate indicator of the true value than any single estimate. But when each bidder sees only one estimate, the largest of these estimates is biased: on average, it will be \$1.08 billion, right near the upper end of the range.[4] Thus, the winning company is likely to pay too much, unless it recognizes the problem and adjusts its bid downward to compensate for this bias. The exact calculation required to determine how

[3] For example, the United States auctions leases for offshore oil-drilling rights, including rights in the Gulf of Mexico and off the coast of Alaska. The state of Pennsylvania auctioned leases for natural gas–drilling rights on almost a quarter of a million acres of state forest land in 2002; this was also the first online, real-time, anonymous auction.

[4] The 10 estimates will, on average, range from \$0.9 billion to \$1.1 billion (\$100 million on either side of \$1 billion). The low and high estimates will, on average, be at the extremes of the distribution.

far to shade down your bid without losing the auction is difficult, however, because you must also recognize that all the other bidders will be making the same adjustment.

We do not pursue the advanced mathematics required to create an optimal bidding strategy in the common-value auction. However, we can provide you with some general advice. If you are bidding on an item, the question "Would I be willing to purchase the lease for $1.08 billion, given what I know before submitting my bid?" is very different from the question "Would I still be willing to purchase the lease for $1.08 billion, given what I know before submitting my bid and given the knowledge that I will be able to purchase the lease only if no one else is willing to bid $1.08 billion for it?"[5] Even in a sealed-bid auction, it is the second question that reveals correct strategic thinking, because you win with any given bid only when all others bid less—only when all other bidders have a lower estimate of the value of the object than you do.

If you do not take the winner's curse into account in your bidding behavior, you should expect to lose substantial amounts, as indicated by the numerical calculations performed above for bidding on the hypothetical Targetco. How real is this danger in practice? Richard Thaler has marshaled a great deal of evidence to show that the danger is very real indeed.[6]

The simplest experiment to test the winner's curse is to auction a jar of pennies. The prize is objective, but each bidder forms a subjective estimate of how many pennies there are in the jar and therefore of the size of the prize; this experiment is a pure example of a common-value auction. Most teachers have conducted such experiments with students and found significant overbidding. In a similar but related experiment, M.B.A. students were asked to bid for a hypothetical company instead of a penny jar. The game was repeated, with feedback after each round on the true value of the company. Only 5 of 69 students learned to bid less over time; the average bid actually went up in the later rounds.

Observations of reality confirm these findings. There is evidence that winners of oil- and gas-drilling leases at auctions take substantial losses on their leases. Baseball players who as free agents went to new teams were found to be overpaid in comparison with those who re-signed with their old teams.

We repeat: The precise calculations that show how much you should shade down your bidding to take into account the winner's curse are beyond the scope of this text; the articles cited in Section 7 contain the necessary mathematical analysis. Here we merely wish to point out the problem and emphasize the need for caution. When your willingness to pay depends on your expected ability to make a profit from your purchase or on the expected resale value of the item, be wary.

[5] See Steven Landsburg, *The Armchair Economist* (New York: Free Press, 1993), p. 175.

[6] Richard Thaler, "Anomalies: The Winner's Curse," *Journal of Economic Perspectives*, vol. 2, no. 1 (Winter 1988), pp. 191–201.

This analysis shows the importance of the prescriptive role of game theory. From observational and experimental evidence, we know that many people fall prey to the winner's curse. By doing so, they lose a lot of money. Learning the basics of game theory would help them anticipate the winner's curse and prevent attendant losses.

3 BIDDING STRATEGIES

We turn now to private-value auctions and a discussion of optimal bidding strategies. Suppose you are interested in purchasing a particular lot of Chateau Margaux 1952 Bordeaux wine. Consider some of the different possible auction procedures that could be used to sell the wine.

A. The English Auction

Suppose first that you are participating in a standard English auction. Your optimal bidding strategy is straightforward, given that you know your valuation V. Start at any step of the bidding process. If the last bid made by a rival bidder—call it r—is at or above V, you are certainly not willing to bid higher; so you need not concern yourself with any further bids. Only if the last bid is still below V do you bid at all. In that case, you can add a penny (or the smallest increment allowed by the auction house) and bid r plus one cent. If the bidding ends there, you get the wine for r (or virtually r), and you make an effective profit of $V - r$. If the bidding continues, you repeat the process, substituting the value of the new last bid for r. In this type of auction, the high bidder gets the wine for (virtually) the valuation of the second-highest bidder. How close the final price is to the second-highest valuation will be determined by the minimum bid increment defined in the auction rules.

B. First-Price, Sealed-Bid, and Dutch Auctions: The Incentive to Shade

Now suppose the wine auction is first price, sealed bid, and you suspect that you are a very high value bidder. You need to decide whether to bid V or something other than V. Should you put in a bid equal to the full value V that you place on the object?

Remember that the high bidder in this auction will be required to pay her bid. In that case, you should not in fact bid V. Such a bid would be sure to give you zero profit, and you could do better by reducing your bid somewhat. If you bid a little less than V, you run the risk of losing the object should a rival bidder make a bid above yours but below V. But as long as you do not bid so low that this outcome is guaranteed, you have a positive probability of making a positive profit. Your optimal bidding strategy entails **shading** your bid. Calculus would be required to describe the actual strategy required here, but an intuitive understanding of the result is simple. An increase in shading (a lowering of your bid from *V*) provides both an advantage and a disadvantage to you; it increases your profit margin if you obtain the wine, but it also lowers your chances of being the high bidder and therefore of actually obtaining the wine. Your bid is optimal when the last bit of shading just balances these two effects.

What about a Dutch auction? Your bidding strategy in this case is similar to that for the first-price, sealed-bid auction. Consider your bidding possibilities. When the price called out by the auctioneer is above *V*, you choose not to bid. If no one has bid by the time the price gets down to *V*, you may choose to do so. But again, as in the sealed-bid case, you have two options. You can bid now and get zero profit or wait for the price to drop lower. Waiting a bit longer will increase the profit that you take from the sale, but it also increases your risk of losing the wine to a rival bidder. Thus, shading is in your interest here as well, and the precise amount of shading depends on the same cost-benefit analysis described in the preceding paragraph.

C. Second-Price, Sealed-Bid Auctions: Vickrey's Truth Serum

Finally, there is the second-price, sealed-bid auction. In that auction, the cost-benefit analysis regarding shading is different from that in the preceding three types of auctions. This result is due to the fact that the advantage gained from shading, the increase in your profit margin, is zero in this auction. You do not improve your profit by shading your bid, because your profit is determined by the second-highest bid, not your own.

From the seller's perspective, this result is encouraging. All else being equal, sellers would prefer bids that were not shaded downward. They are thus faced with a problem in mechanism design in which they want to induce information revelation—induce bidders to reveal their true valuations with their bids.

William Vickrey showed that truthful revelation of valuations from bidders would arise if the seller of a private-value object used a modified version of the standard, first-price, sealed-bid scheme; his suggestion was to modify the sealed-bid auction so that it more closely resembles its open-outcry counterpart.[7] That is, the highest bidder should get the object for a price equal to the second-highest bid—a second-price, sealed-bid auction. Vickrey showed that, with these rules, every bidder has a dominant bidding strategy to bid her true valuation. Thus, we facetiously dub it **Vickrey's truth serum**.

[7] Vickrey was one of the most original minds in economics in the past four decades. In 1996, he won the Nobel Prize for his work on mechanism design in auctions and truth-revealing procedures. Sadly, he died just 3 days after the prize was announced.

However, we saw in Chapter 13 that there is usually a cost to using a mechanism that extracts information. Auctions are no exception. Buyers reveal the truth about their valuations in an auction using Vickrey's scheme only because it gives them some profit from doing so. The second-price, sealed-bid auction mechanism reduces the profit for the seller, just as the shading of bids does in a first-price auction, and just as the information-revelation mechanisms we studied in Chapter 13 did for the principals in those cases. The relative merit of the two procedures from the seller's point of view therefore depends on which one entails a greater reduction in her profit. We consider this matter later in Section 5; but first we explain how Vickrey's scheme works.

Suppose you are an antique-china collector, and you have discovered that a local estate auction will be selling off a nineteenth-century Meissen "Blue Onion" tea set in a sealed-bid, second-price auction. As someone experienced with vintage china but lacking this set for your collection, you value it at \$3,000, but you do not know the valuations of the other bidders. If they are inexperienced, they may not realize the considerable value of the set. If they have sentimental attachments to Meissen or the "Blue Onion" pattern, they may value it more highly than the value that you have calculated.

The rules of the auction allow you to bid any real-dollar value for the tea set. We will call your bid b and consider all of its possible values. Because you are not constrained to a small, specific set of bids, we cannot draw a finite payoff matrix for this bidding game, but we can logically deduce the optimal bid.

The success of your bid will obviously depend on the bids submitted by others interested in the tea set, because you need to consider whether your bid can win. The outcome thus depends on all rival bids, but only the largest bid among them will affect your outcome. We call this largest rival bid r and disregard all bids below r.

What is your optimal value of b? We will look at bids both above and below \$3,000 to determine whether any option other than exactly \$3,000 can yield you a better outcome than bidding your true valuation.

We start with $b > 3{,}000$. There are three cases to consider. First, if your rival bids less than \$3,000 (so $r < 3{,}000$), then you get the tea set at the price r. Your profit, which depends only on what you pay relative to your true valuation, is $(3{,}000 - r)$, which is what it would have been had you simply bid \$3,000. Second, if your rival's bid falls between your actual bid and your true valuation (so $3{,}000 < r < b$), then you are forced to take the tea set for more than it is worth to you. Here you would have done better to bid \$3,000; you would not have gotten the tea set, but you would not have given up the $(r - 3{,}000)$ in lost profit either. Third, your rival bids even more than you do (so $b < r$). You still do not get the tea set, but you would not have gotten it even had you bid your true valuation. Putting together the reasoning of the three cases, we see that bidding your true valuation is never worse, and sometimes better, than bidding something higher.

What about the possibility of shading your bid slightly and bidding $b <$ 3,000? Again, there are three situations. First, if your rival's bid is lower than yours (so $r < b$), then you are the high bidder, and you get the tea set for r. Here you could have gotten the same result by bidding $3,000. Second, if your rival's bid falls between 3,000 and your actual bid (so $b < r < 3{,}000$), your rival gets the tea set. If you had bid $3,000 in this case, you would have gotten the tea set, paid r, and still made a profit of $(3{,}000 - r)$. Third, your rival's bid could have been higher than $3,000 (so $3{,}000 < r$). Again, you do not get the tea set but, if you had bid $3,000, you still would not have gotten it, so there would have been no harm in doing so. Again, we see that bidding your true valuation, then, is no worse, and sometimes better, than bidding something lower.

If truthful bidding is never worse and sometimes better than bidding either above or below your true valuation, then you do best to bid truthfully. That is, no matter what your rival bids, it is always in your best interest to be truthful. Put another way, bidding your true valuation is your dominant strategy whether you are allowed discrete or continuous bids.

Vickrey's remarkable result that truthful bidding is a dominant strategy in second-price, sealed-bid auctions has many other applications. For example, if each member of a group is asked what she would be willing to pay for a public project that will benefit the whole group, each has an incentive to understate her own contribution—to become a "free rider" on the contributions of the rest. We have already seen examples of such effects in the collective-action games of Chapter 11. A variant of the Vickrey scheme can elicit the truth in such games as well.

4 ALL-PAY AUCTIONS

We have considered most of the standard auction types discussed in Section 1 but none of the more creative configurations that might arise. Here we consider a common-value, sealed-bid, first-price auction in which every bidder, win or lose, pays to the auctioneer the amount of her bid. An auction where the losers also pay may seem strange. But in fact, many contests result in this type of outcome. In political contests, all candidates spend a lot of their own money and a lot of time and effort for fund raising and campaigning. The losers do not get any refunds on all their expenditures. Similarly, hundreds of competitors spend four years of their lives preparing for an event at the next Olympic games. Only one wins the gold medal and the attendant fame and endorsements; two others win the far less valuable silver and bronze medals; the efforts of the rest are wasted.

The tournaments we discussed in Chapter 13, Section 6.B, are similar. Once you start thinking along these lines, you will realize that such all-pay auctions are, if anything, more frequent in real life than situations resembling the standard formal auctions where only the winner pays.

How should you bid (that is, what should your strategy be for expenditure of time, effort, and money) in an **all-pay auction**? Once you decide to participate, your bid is wasted unless you win, so you have a strong incentive to bid very aggressively. In experiments, the sum of all the bids often exceeds the value of the prize by a large amount, and the auctioneer makes a handsome profit.[8] In that case, everyone's submitting extremely aggressive bids cannot be the equilibrium outcome; it seems wiser to stay out of such destructive competition altogether. But if everyone else did that, then one bidder could walk away with the prize for next to nothing; thus, not bidding cannot be an equilibrium strategy either. This analysis suggests that the equilibrium lies in mixed strategies.

Consider a specific auction with n bidders. To keep the notation simple, we choose units of measurement so that the common-value object (prize) is worth 1. Bidding more than 1 is sure to bring a loss, and so we restrict bids to those between 0 and 1. It is easier to let the bid be a continuous variable x, where x can take on any (real) value in the interval $[0, 1]$. Because the equilibrium will be in mixed strategies, each person's bid, x, will be a continuous random variable. Because you win the object only if all other bidders submit bids below yours, we can express your equilibrium mixed strategy as $P(x)$, the probability that your bid takes on a value less than x; for example, $P(1/2) = 0.25$ would mean that your equilibrium strategy entailed bids below 1/2 one-quarter of the time (and bids above 1/2 three-quarters of the time).[9]

As usual, we can find the mixed-strategy equilibrium by using an indifference condition. Each bidder must be indifferent about the choice of any particular value of x, given that the others are playing their equilibrium mixes. Suppose you, as one of the n bidders, bid x. You win if all of the remaining $(n - 1)$ are bidding less than x. The probability of anyone else bidding less than x is $P(x)$; the probability of two others bidding less than x is $P(x) \times P(x)$, or $[P(x)]^2$; the probability of all $(n - 1)$ of them bidding less than x is $P(x) \times P(x) \times P(x) \ldots$ multiplied $(n - 1)$ times, or $[P(x)]^{n-1}$. Thus with a probability of $[P(x)]^{n-1}$, you win 1. Remember that you pay x no matter what happens. Therefore, your net

[8] One of us (Dixit) has auctioned \$10 bills to his Games of Strategy class and made a profit of as much as \$60 from a 20-student section. At Princeton there is a tradition of giving the professor a polite round of applause at the end of a semester. Once Dixit offered \$20 to the student who kept applauding continuously the longest. This is an open-outcry, all-pay auction with payments in kind (applause). Although most students dropped out between 5 and 20 minutes, three went on for 4½ hours!

[9] $P(x)$ is called the *cumulative probability distribution function* for the random variable x. The more familiar probability density function for x is its derivative, $P'(x) = p(x)$. Then $p(x)\,dx$ denotes the probability that the variable takes on a value in a small interval from x to $x + dx$.

expected payoff for any bid of x is $[P(x)]^{n-1} - x$. But you could get 0 for sure by bidding 0. Thus, because you must be indifferent about the choice of any particular x, including 0, the condition that defines the equilibrium is $[P(x)]^{n-1} - x = 0$. In a full mixed-strategy equilibrium, this condition must be true for all x. Therefore, the equilibrium mixed-strategy bid is $P(x) = x^{1/(n-1)}$.

A couple of sample calculations will illustrate what is implied here. First, consider the case in which $n = 2$; then $P(x) = x$ for all x. Therefore, the probability of bidding a number between two given levels x_1 and x_2 is $P(x_2) - P(x_1) = x_2 - x_1$. Because the probability that the bid lies in any range is simply the length of that range, any one bid must be just as likely as any other bid. That is, your equilibrium mixed-strategy bid should be random and uniformly distributed over the whole range from 0 to 1.

Next let $n = 3$. Then $P(x) = \sqrt{x}$. For $x = 1/4$, $P(x) = 1/2$; so the probability of bidding 1/4 or less is 1/2. The bids are no longer uniformly distributed over the range from 0 to 1; they are more likely to be in the lower end of the range.

Further increases in n reinforce this tendency. For example, if $n = 10$, then $P(x) = x^{1/9}$, and $P(x)$ equals 1/2 when $x = (1/2)^9 = 1/512 = 0.00195$. In this situation, your bid is as likely to be smaller than 0.00195 as it is to be anywhere within the whole range from 0.00195 to 1. Thus, your bids are likely to be very close to 0.

Your average bid should correspondingly be smaller the larger the number n. In fact, a more precise mathematical calculation shows that if everyone bids according to this strategy, the average or expected bid of any one player will be just $(1/n)$.[10] With n players bidding, on average, $1/n$ each, the total expected bid is 1, and the auctioneer makes zero expected profit. This calculation provides more precise confirmation that the equilibrium strategy eliminates overbidding.

The idea that your bid should be much more likely to be close to 0 when the total number of bidders is large makes excellent intuitive sense, and the finding that equilibrium bidding eliminates overbidding lends further confidence to the theoretical analysis. Unfortunately, many people in actual all-pay auctions either do not know or forget this theory and bid to excess.

Interestingly, philanthropists have figured out how to take this tendency to overbid and harness it for social benefit. Building on the historical lessons learned from prizes offered in 1919 by a New York hotelier for the first nonstop transatlantic flight (won by Charles Lindbergh in 1927) and even earlier, in 1714, by the British government for a method to precisely measure longitude for sea navigation (eventually awarded to John Harrison in the 1770s), several U.S. and international foundations have begun offering incentive prizes for various

[10] The expected bid of any one player is calculated as the expected value of x, by using the probability density function, $p(x)$. In this case, $p(x) = P'(x) = [1/(n-1)]x^{(2-n)/(n-1)}$, and the expected value of x is the sum, or integral, of this from 0 to 1, namely $\int x\, p(x)\, dx = 1/n$.

socially worthwhile innovations. One foundation in particular, the X Prize Foundation, has as its sole purpose the provision of incentive prizes; its first prize was awarded in 2004 for the first private space flight. Twenty-two teams are now in competition for a $30 million prize for the first landing of a robot on the moon. These teams have until the end of 2015 to claim the prize. Some foundation experts estimate that as much as 40 times the amount of money that would otherwise be devoted to a particular innovation gets spent when incentive prizes are available. Thus, the tendency to overbid in all-pay auctions can actually have a beneficial impact on society (if not on the individual pursuing the prize).[11]

5 HOW TO SELL AT AUCTION

Bidders are not the only auction participants who need to consider their optimal strategies carefully. An auction is really a sequential-play game in which the first move is the setting of the rules; bidding starts only in the second round of moves. It falls to the sellers, then, to determine the path that later bidding will follow by choosing a particular auction rule or mechanism.

As a seller interested in auctioning off your prized art collection or even your home, you must decide on the best auction mechanism or rule to use. To guarantee yourself the greatest profit from your sale, you must look ahead to the predicted outcome of the different auction mechanisms before making a choice. One concern of many sellers is that an item will go to a bidder for a price lower than the value that the seller places on the object. To counter this concern, most sellers insist on setting a **reserve price** for auctioned objects; they reserve the right to withdraw the object from the sale if no bid higher than the reserve price is obtained.

Beyond setting a reserve price, however, what can sellers do to determine the type of auction mechanism that might net them the most profit possible? One possibility is to use Vickrey's suggested scheme, a second-price, sealed-bid auction. According to him, this kind of auction elicits truthful bidding from potential buyers. Does this effect make it a good auction type from the seller's perspective?

In a sense, the seller in such a second-price auction is giving the bidder a profit margin to counter the temptation to shade down the bid in the hope of a larger profit. But this outcome then reduces the seller's revenue, just as shading

[11] For more on incentive prizes, see Matthew Leerberg, "Incentivizing Prizes: How Foundations Can Utilize Prizes to Generate Solutions to the Most Intractable Social Problems," Duke University Center for the Study of Philanthropy and Voluntarism Working Paper, Spring 2006. Information on the X Prize Foundation is available at www.xprize.org.

down in a first-price, sealed-bid auction would. Which type of auction mechanism is ultimately better for the seller actually turns out to depend on the bidders' attitudes toward risk and their beliefs about the value of the object for sale. The relative merits of different mechanisms in reality can also depend on other issues such as the possibility of collusion among bidders, and the choice can also involve political considerations when selling public property such as the airwave spectrum or drilling rights. Thus, the auction environment is critical to seller revenue.[12]

A. Risk-Neutral Bidders and Independent Estimates

The least complex configuration of bidder risk attitudes and beliefs occurs when there is risk neutrality (no risk aversion) and when bidder estimates about the value of the object for sale remain independent of each other. As we said in the appendix to Chapter 8, risk-neutral people care only about the expected monetary value of their outcomes, regardless of the level of uncertainty associated with those outcomes. Independence in estimates means that a bidder is not influenced by the estimates of other bidders when determining how much an object is worth to her; the bidder has decided independently exactly how much the object is worth to her. In this case, there can be no winner's curse. If these conditions for bidders hold, sellers can expect the same average revenue (over a large number of trials) from any of the four primary types of auction: English, Dutch, and first- and second-price sealed-bid.

This *revenue equivalence* result implies not that all of the auctions will yield the same revenue for every item sold, but that the auctions will yield the same selling price on average in the course of numerous auctions. We have already seen that, in the second-price auction, each bidder's dominant strategy is to bid her true valuation. The highest bidder gets the object for the second-highest bid, and the seller gets a price equal to the valuation of the second-highest bidder. Similarly, in an English auction, bidders drop out as the price increases beyond their valuations, until only the first- and second-highest-valuation bidders remain. When the price reaches the valuation of the second-highest bidder, that bidder also will drop out, and the remaining (highest-valuation) bidder will take the object for just a cent more than the second-highest bid. Again, the seller gets a price (essentially) equivalent to the valuation of the second-highest bidder.

More advanced mathematical techniques are needed to prove that revenue equivalence can be extended to Dutch and first-price, sealed-bid auctions as

[12] Klemperer's book, especially chapters 3 and 4, has detailed discussions and warnings on all these issues.

well, but the intuition should be clear. In all four types of auctions, in the absence of any risk aversion on the part of bidders, the highest-valuation bidder should win the auction and pay on average a price equal to the second-highest valuation. If the seller is likely to use a particular auction mechanism repeatedly, she need not be overly concerned about her choice of auction structure; all four would yield her the same expected price.

Experimental and field evidence has been collected to test the validity of the revenue-equivalent theorem in actual auctions. The results of laboratory experiments tend to show Dutch auction prices lower, on average, than first-price, sealed-bid auction prices for the same items being bid on by the same group of bidders, possibly owing to some positive utility associated with the suspense factor in Dutch auctions. These experiments also find evidence of overbidding (bidding above your known valuation) in second-price, sealed-bid auctions but not in English auctions. Such behavior suggests that bidders go higher when they have to specify a price, as they do in sealed-bid auctions; these auctions seem to draw more attention to the relationship between the bid price and the probability of ultimately winning the item. Field evidence from Internet auctions finds literally opposite results, with Dutch auction revenue as much as 30% higher, on average, than first-price, sealed-bid revenue. Additional bidder interest in the Dutch auctions or impatience in the course of a 5-day auction could explain the anomaly. The Internet-based field evidence did find near revenue equivalence for the other two auction types.

B. Risk-Averse Bidders

Here we continue to assume that bids and beliefs are uncorrelated but incorporate the possibility that auction outcomes could be affected by bidders' attitudes toward risk. In particular, suppose bidders are risk averse. They may be much more concerned, for example, about the losses caused by underbidding—losing the object—than by the costs associated with bidding at or close to their true valuations. Thus, risk-averse bidders generally want to win if possible without ever overbidding.

What does this preference structure do to the types of bids that they submit in first-price versus second-price (sealed-bid) auctions? Again, think of the first-price auction as being equivalent to the Dutch auction. Here, risk aversion leads bidders to bid earlier rather than later. As the price drops to the bidder's valuation and beyond, there is greater and greater risk in waiting to bid. We expect risk-averse bidders to bid quickly, not to wait just a little bit longer in the hope of gaining those extra few pennies of profit. Applying this reasoning to the first-price, sealed-bid auction, we expect bidders to shade down their bids by less than they would if they were not risk averse: too much shading actually increases the risk of not gaining the object, which risk-averse bidders would want to avoid.

Compare this outcome with that of the second-price auction, where bidders pay a price equal to the second-highest bid. Bidders bid their true valuations in such an auction but pay a price less than that. If they shade their bids only slightly in the first-price auction, then those bids will tend to be close to the bidders' true valuations—and bidders pay their bids in such auctions. Thus, bids will be shaded somewhat, but the price ultimately paid in the first-price auction will probably exceed what would be paid in the second-price auction. When bidders are risk averse, the seller then does better to choose a first-price auction rather than a second-price auction.

The seller does better with the first-price auction in the presence of risk aversion only in the sealed-bid case. If the auction is English, the bidders' attitudes toward risk are irrelevant to the outcome. Thus, risk aversion does not alter the outcome for the seller in these auctions.

C. Correlated Estimates

Now suppose that in determining their own valuations of an object, bidders are influenced by the estimates (or by their beliefs about the estimates) of other bidders. Such a situation is relevant for common-value auctions, such as those for oil or gas exploration considered in Section 2. Suppose your experts have not presented a glowing picture of the future profits to be gleaned from the lease on a specific tract of land. You are therefore pessimistic about its potential benefits, and you have constructed an estimate V of its value that you believe corresponds to your pessimism.

Under the circumstances, you may be concerned that your rival bidders also have received negative reports from their experts. When bidders believe their valuations are all likely to be similar, either all relatively low or all relatively high, for example, we say that those beliefs or estimates of value are positively correlated. Thus, the likelihood that your rivals' estimates also are unfavorable may magnify the effect of your pessimism on your own valuation. If you are participating in a first-price, sealed-bid auction, you may be tempted to shade down your bid even more than you would in the absence of correlated beliefs. If bidders are optimistic and valuations generally high, correlated estimates may lead to less shading than when estimates are independent.

However, the increase in the shading of bids that accompanies correlated low (or pessimistic) bids in a first-price auction should be a warning to sellers. With positively correlated bidder beliefs, the seller may want to avoid the first-price auction and take advantage of Vickrey's recommendation to use a second-price structure. We know that this auction mechanism encourages truthful revelation, and when correlated estimates are possible, the seller does even better to avoid auctions in which there might be any additional shading of bids.

An English auction will have the same ultimate outcome as the second-price, sealed-bid auction, and a Dutch auction will have the same outcome as a first-price, sealed-bid auction. Thus, a seller facing bidders with correlated estimates of an object's value also should prefer the English to the Dutch version of the open-outcry auction. If you are bidding on the oil-land lease in an English auction and the price is nearing your estimate of the lease's value but your rivals are still bidding feverishly, you can infer that their estimates are at least as high as yours—perhaps significantly higher. The information that you obtain from observing the bidding behavior of your rivals may convince you that your estimate is too low. You might even increase your own estimate of the land's value as a result of the bidding process. Your continuing to bid may provide an impetus for further bidding by other bidders, and the process may continue for a while. If so, the seller reaps the benefits. More generally, the seller can expect a higher selling price in an English auction than in a first-price, sealed-bid auction when bidder estimates are correlated. For the bidders, however, the effect of the open bidding is to disperse additional information and to reduce the effect of the winner's curse.

The discussion of correlated estimates assumes that a fairly large number of bidders take part in the auction. But an English auction can be beneficial to the seller if there are only two bidders, both of whom are particularly enthusiastic about the object for sale. They will bid against each other as long as possible, pushing the price up to the lower of the valuations, both of which were high from the start. The same auction can be disastrous for the seller, however, if one of the bidders has a very low valuation; the other is then quite likely to have a valuation considerably higher than the first. In this case, we say that bidder valuations are negatively correlated. We encourage any seller facing a small number of bidders with potentially very different valuations to choose a Dutch or first-price, sealed-bid structure. Either of them would reduce the possibility of the high-valuation bidder gaining the object for well under her true valuation; that is, either type would transfer the available profit from the buyer to the seller.

6 SOME ADDED TWISTS TO CONSIDER

A. Multiple Objects

When you think about an auction of a group of items, such as a bank's auctioning repossessed vehicles or estate sales auctioning the contents of a home, you probably envision the auctioneer bringing each item to the podium individually and selling it to the highest bidder. This process is appropriate when each bidder has independent valuations for each item. However, independent valuations may not always be an appropriate way to model bidder estimates. Then, if bidders value specific groups or whole packages of items higher than the sum of their values for the component items, the choice of auctioning the lots separately or together makes a big difference to bidding strategies as well as to outcomes.

Consider a real-estate developer named Red who is interested in buying a very large parcel of land on which to build a townhouse community for professionals. Two townships, Cottage and Mansion, are each auctioning a land parcel big enough to suit her needs. Both parcels are essentially square in shape and encompass 4 square acres. The mayor of Cottage has directed that the auctioneer sell the land as quarter-acre blocks, one at a time, starting at the perimeter of the land and working inward, selling the corner lots first and then the lots on the north, south, east, and west borders in that order. At the same time, the mayor of Mansion has directed that the auctioneer attempt to sell the land in her town first as a full 4-acre block, then as two individual 2-acre lots, and then as four 1-acre lots after that, if no bids exceed the set reserve prices.

Through extensive market analyses, Red has determined that the blocks of land in Cottage and Mansion would provide the same value to her. However, she has to obtain the full 4 acres of land in either town to have enough room for her planned development. The auctions are being held on the same day at the same time. Which should she attend?

It should be clear that her chances of acquiring a 4-acre block of land for a reasonable price—less than or equal to her valuation—are much better in Mansion than in Cottage. In the Mansion auction, she would simply wait to see how bidding proceeded, submitting a final high bid if the second-highest offer fell below her valuation of the property. In the Cottage auction, she would need to win each and every one of the 16 parcels up for sale. Under the circumstances, she should expect rival bidders interested in owning land in Cottage to become more intent on their goals—perhaps even joining forces—as the number of available parcels decreases in the course of the auction. Red would have to bid aggressively enough to win parcels in the early rounds while being conservative enough to ensure that she did not exceed her total valuation by the end of the auction. The difficulties in crafting a bidding strategy for such an auction are numerous, and the probability of being unable to obtain every parcel profitably is quite large—hence Red's preference for the Mansion auction.

Note that, from the seller's point of view, the Cottage auction is likely to bring in greater revenue than the Mansion auction if an adequate number of bidders are interested in small pieces of land. If the only bidders are all developers like Red, however, they might be hesitant even to participate in the Cottage auction for fear of being beaten in just one round. In that case, the Mansion-type auction mechanism is better for the seller.

The township of Cottage could allay the fears of developers by revising the rules for its auction. In particular, it would not need to auction each parcel individually. Instead it could hold a single auction in which all parcels would be available simultaneously. Such an auction could be run so that each bidder could specify the number of parcels that she wanted and the price that she was willing to pay per parcel. The bidder with the highest total-value bid—determined by multiplying the number of parcels desired by the price for each—would win the desired number of parcels. If parcels remained after the high bidder took her land, additional parcels would be won in a similar way until all the land was sold. This mechanism gives bidders interested in larger parcels an opportunity to bid, potentially against each other, for blocks of the land. Thus, Cottage might find this type of auction more lucrative in the end.

B. Defeating the System

We saw earlier which auction mechanism is best for the seller, given different assumptions about how bidders felt toward risk and whether their estimates were correlated. There is always an incentive for bidders, though, to come up with a bidding strategy that defeats the seller's efforts. The best-laid plans for a profitable auction can almost always be defeated by an appropriately clever bidder or, more often, group of bidders.

Even the Vickrey second-price, sealed-bid auction can be defeated if there are only a few bidders in the auction, all of whom can collude among themselves. By submitting one high bid and a lowball second-highest bid, collusive bidders can obtain an object for the second-bid price. This outcome results only if no other bidders submit intermediate bids or if the collusive bidders are able to prevent such an occurrence. The possibility of collusion highlights the need for the seller's reserve prices, although they only partly offset the problem in this case.

First-price, sealed-bid auctions are less vulnerable to bidder collusion for two reasons. The potential collusive group engages in a multiperson prisoners' dilemma game in which each bidder has a temptation to cheat. In such cheating, an individual bidder might submit her own high bid so as to win the object for herself, reneging on any obligation to share profits with group members. Collusion among bidders in this type of auction is also difficult to sustain because cheating (that is, making a different bid from that agreed to within the collusive group) is easy to do but difficult for other buyers to detect. Thus, the sealed-bid nature of the auction prevents detection of a cheater's behavior, and hence punishment, until the bids are opened and the auction results announced; at that point, it is simply too late. However, there may be more scope for sustaining collusion if a particular group of bidders participates in a number of similar auctions over time, so that they engage in the equivalent of a repeated game.

Other tricky bidding schemes can be created to meet the needs of specific individual bidders or groups of bidders in any particular type of auction. One very clever example of bid rigging arose in an early U.S. Federal Communications Commission auction of the U.S. airwave spectrum, specifically for personal cellular service (Auction 11, August 1996–January 1997). After watching prices soar in some of the earlier auctions, bidders were apparently eager to reduce the price of the winning bids. The solution, used by three firms (later sued by the Department of Justice), was to signal their intentions to go after licenses for certain geographic locations by using the FCC codes or telephone area codes for those areas as the last three digits of their bids. The FCC has claimed that this practice significantly reduced the final prices on these particular licenses. In addition, other signaling devices were apparently used in earlier broadband auctions. While some firms literally announced their intentions to win a particular license, others used a variety of strategic bidding techniques to signal their interest in specific licenses or to dissuade rivals from horning in on their territories. In the first broadband auction, for example, GTE and other firms apparently used the code-bidding technique of ending their bids with the numbers that spelled out their names on a telephone keypad!

We note briefly here that fraudulent behavior is not merely the territory of bidders at auction. Sellers also can use underhanded practices to inflate the final bid price of pieces that they are attempting to auction. **Shilling**, for example, occurs when a seller is able to plant false bids at her own auction. Possible only in English auctions, shilling can be done with the use of an agent who works for the seller and who pretends to be a regular bidder. On Internet auction sites, shilling is actually easier, because a seller can register a second identity and log in and bid in her own auction; all Internet auctions have rules and oversight mechanisms designed to prevent such behavior. Sellers in second-price, sealed-bid auctions can also benefit if they inflate the level of the (not publicly known) second-highest bid.

C. Information Disclosure

Finally, we consider the possibility that the seller has some private information about an object that might affect the bidders' valuations of that object. Such a situation arises when the quality or durability of a particular object, such as an automobile, a house, or a piece of electronic equipment, is of great importance to the buyers. Then the seller's past experience with the object may be a good predictor of the future benefits that will accrue to the winning bidder.

As we saw in Chapter 8, the more informed player in an asymmetric information game must decide whether to reveal or conceal her private information. In the auction context, a seller must carefully consider any temptation to conceal information. If the bidders know that the seller has some private information, they are likely to interpret any failure to disclose that information as a signal that the information is unfavorable. Even if the seller's information is unfavorable, she may be better off revealing it; bidders' beliefs might be worse than the actual information. Thus, honesty is often the best policy.

Honesty can also be in the seller's interest for another reason. When she has private information about a common-value object, she should disclose that information to sharpen the bidders' estimates of the value of the object. The more confident the bidders are that their valuations are correct, the more likely they are to bid up to those valuations. Thus, disclosure of private seller information in a common-value auction can help not only the seller by reducing the amount of shading done by bidders but also the bidders by reducing the effects of the winner's curse.

D. Online Auctions

Internet auction sites have been in existence for almost two decades. The eBay site began operation in September 1995, shortly after the advent of Onsale.com in May of that year.[13] A large number of auction sites, approximately 100, now exist; precise numbers change frequently as new sites are created, as mergers are consummated between existing sites, and as smaller, unprofitable sites shut down. These sites, both small and large, sell an enormous variety of items in many different ways.

The majority of auction items on the larger sites, such as eBay and uBid, are goods that are classified as "collectibles." There are also specialty auction sites that deal with items ranging from postage stamps, wine, and cigars to seized property from police raids, medical equipment, and large construction equipment (scissorlift, anyone?). Most of these items, regardless of the type of site, would be considered "used." Thus, consumers have access to what might be called the world's largest garage sale, all at their fingertips. This information is consistent with one hypothesis in the literature that suggests that Internet auctions are most useful for selling goods available in limited quantity, for which there is unknown demand, and for which the seller cannot easily determine an appropriate price. The auction process can effectively find a "market price" for these goods. Sellers of such goods then have their best profit opportunity online, where a broad audience can supply formerly unknown demand parameters. And consumers can obtain desired but obscure items, presumably with profit margins of their own.

[13] Onsale merged with Egghead.com in 1999. Amazon bought the assets of the merged company late in 2001. The three auction sites originally available as Onsale, Egghead, and Amazon became Amazon Auctions. These have been replaced by Amazon's fixed-price selling outlet, Marketplace.

In addition to selling many different categories of goods, Internet auctions employ a variety of auction rules. Many sites actually offer several auction types and allow a seller to choose her auction's rules when she lists an item for sale. The most commonly used rules are those for English and second-price, sealed-bid auctions; one or both of them are offered by the majority of auction sites.

Sites that offer true English auctions post the high bid as soon as it is received; at the end of the auction, the winner pays her bid. Others that appear to use the English auction format allow what is known as **proxy bidding**. The proxy-bidding process actually makes the auction second-price, sealed-bid rather than English. With proxy bidding, a bidder enters the maximum price that she is willing to pay (her **reservation price**) for an item. Rather than displaying this maximum price, the auction site displays only one bid increment above the most recent high bid. The proxy-bidding system then bids for the buyer, outbidding others by a single bid increment, until the buyer's maximum price is reached. This system allows the auction winner to pay just one bid increment over the second-highest bid rather than paying her own bid.

Dutch auction formats at online auction sites are quite rare. Only a very few retail sites now offer the equivalent of Dutch auctions. Lands' End, for example, posts some overstocked items each weekend in a special area of its Web site; it then reduces the prices on these items three times in the next week, removing unsold items at week's end. Some sites offer auctions *called* Dutch auctions that, along with a companion type known as **Yankee auctions**, actually offer multiple (identical) units in a single auction. Similar to the auction described above for the available land parcels in the township of Cottage, these auctions offer bidders the option to bid on one or more of the units. The terminology "Yankee auction" refers to the system that we described for the Cottage auction; bidders with the highest total-value bid(s) win the items, and each bidder pays her bid price per unit. The "Dutch auction" label is reserved for auctions in which bids are ranked by total value, but at the end of the auction, all bidders pay the lowest winning bid price for their units.[14]

The Internet has also made it possible to create and apply auction rules that would previously have been impractical. The newest such auction is one in which the *lowest unmatched bid* is the one that wins the item; the object trades at the winning bid price. How can a seller afford such an auction? She can do so simply by auctioning a fairly valuable item, such as a piece of real estate or a sizeable quantity of gold bullion, and charging a small fee for each bid. Bidding continues until a specified number of bids is obtained, at which time the lowest unmatched bid is awarded the object. The success of this type of online auction remains to be seen. One initially successful site, humraz.com, has been shut

[14] This Dutch-type mechanism is also used by the Federal Reserve to auction Treasury bills.

down. Others, such as winnit.com, continue to be profitable, but these so-called "penny auctions" have not caught on with the general public.

Although the Internet does allow for creativity, most Internet auctions tend to be quite similar in their rules and outcomes to traditional live auctions. Strategic issues such as those considered earlier in this chapter are relevant to both. There are some benefits to online auctions as well as costs. Online auctions are good for buyers because they are easy to "attend" and they provide search engines that make it simple to identify items of interest. Similarly, sellers are able to reach a wide audience and often have the convenience of choosing the rules of their own auctions. Many U.S. counties now enlist the services of RealAuction.com to provide online access to their formerly live foreclosure and tax lien auctions. However, online auction sales can suffer from the fact that buyers cannot inspect goods before they must bid and because buyer and seller must each trust the other to pay or deliver as promised.

The most interesting difference between live and online auctions, though, is the way in which the auctions must end. Live (English and Dutch) auctions cease when no additional bids can be obtained. Online auctions need to have specific auction-ending rules. The two most commonly used rules specify either a fixed moment in time or a certain number of minutes beyond the most recent bid (after a predetermined amount of time has passed). Evidence has been gathered by Alvin Roth and Axel Ockenfels that shows that hard end times make it profitable for bidders to bid late. This behavior, referred to as *sniping*, is found in both private-value and common-value auctions. Strategically, such late bidders gain by avoiding bidding wars with others who update their own bids throughout the auction. In addition, they gain by protecting any private information that they hold regarding the common valuation of a good. These advantages are not available in auctions with end-time extensions where bidders can more safely make a single proxy bid at any time during the auction.

As of late 2014, the initial popularity of online auction sites for the sale of "used" goods has declined precipitously. Although eBay continues to be a treasure trove for online shoppers, the percentage of items available by auction-only mechanisms on that site dropped from greater than 95% in the beginning of 2003 to just under 15% in early 2012. Auction-style sales have been replaced by fixed-price items and "buy it now" options on many auction sites. Recent research into this phenomenon suggests it can be attributed to shifts in buyer preferences away from the riskier feeling, and more time intensive, auction mechanisms in favor of a more traditional consumer experience.[15]

[15] For more on the demise of the auction mechanism in online sales, see Liran Einav, Chiara Farronato, Jonathan D. Levin, and Neel Sundaresan, "Sales Mechanisms in Online Markets: What Happened to Internet Auctions?" NBER Working Paper No. 19021, May 2013.

7 ADDITIONAL READING

Much of the literature on the theory of auctions is quite mathematically complex. Some general insights into auction behavior and outcomes can be found in Paul Milgrom, "Auctions and Bidding: A Primer"; Orley Ashenfelter, "How Auctions Work for Wine and Art"; and John G. Riley, "Expected Revenues from Open and Sealed Bid Auctions," all in the *Journal of Economic Perspectives*, vol. 3, no. 3 (Summer 1989), pp. 3–50. These papers should be readable by those of you with a reasonably strong background in calculus.

More complex information on the subject also is available. R. Preston McAfee and John McMillan have an overview paper, "Auctions and Bidding," in the *Journal of Economic Literature*, vol. 25 (June 1987), pp. 699–738. A more recent review of the literature can be found in Paul Klemperer, "Auction Theory: A Guide to the Literature," in the *Journal of Economic Surveys*, vol. 13, no. 3 (July 1999), pp. 227–286. Both of these pieces contain some of the high-level mathematics associated with auction theory but also give comprehensive references to the rest of the literature. Klemperer's book *Auctions: Theory and Practice* (Princeton: Princeton University Press, 2004) has a more recent and somewhat less mathematically complex survey in chapter 1.

Vickrey's original article containing the details on truthful bidding in second-price auctions is "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, vol. 16, no. 1 (March 1961), pp. 8–37. This paper was one of the first to note the existence of revenue equivalence. A more recent study gathering a number of the results on revenue outcomes for various auction types is J. G. Riley and W. F. Samuelson, "Optimal Auctions," *American Economic Review*, vol. 71, no. 3 (June 1981), pp. 381–392. A very readable history of the "Vickrey" second-price auction is David Lucking-Reiley, "Vickrey Auctions in Practice: From Nineteenth-Century Philately to Twenty-First-Century E-Commerce," *Journal of Economic Perspectives*, vol. 14, no. 3 (Summer 2000), pp. 183–192.

Some of the experimental evidence on auction behavior is reviewed in John H. Kagel, "Auctions: A Survey of Experimental Research," in *The Handbook of Experimental Economics*, ed. John Kagel and Alvin Roth (Princeton: Princeton University Press, 1995), pp. 501–535, and in a companion piece with Dan Levin, "Auctions: A Survey of Experimental Research, 1995–2007," forthcoming in the handbook's second volume. Other evidence on behavior in online auctions is presented in Alvin Roth and Axel Ockenfels, "Late and Multiple Bidding in Second-Price Internet Auctions: Theory and Evidence Concerning Different Rules for Ending an Auction," *Games and Economic Behavior*, vol. 55, no. 2 (May 2006), pp. 297–320.

For information specific to auction design, see Paul Klemperer, "What Really Matters in Auction Design," *Journal of Economic Perspectives*, vol. 16, no. 1 (Winter 2002), pp. 169–189. A survey of Internet auctions can be found in David Lucking-Reiley, "Auctions on the Internet: What's Being Auctioned, and How?" *Journal of Industrial Economics*, vol. 48, no. 3 (September 2000), pp. 227–252.

SUMMARY

In addition to the standard *first-price, open-outcry, ascending,* or *English* auction, there are also *Dutch*, or *descending*, auctions as well as *first-price* and *second-price, sealed-bid* auctions. Objects for bid may have a single *common value* or many *private values* specific to each bidder. With common-value auctions, bidders often win only when they have overbid, falling prey to the *winner's curse*. In private-value auctions, optimal bidding strategies, including decisions about when to *shade* down bids from your true valuation, depend on the auction type used. In the familiar first-price auction, there is a strategic incentive to underbid.

Vickrey showed that sellers can elicit true valuations from bidders by using a second-price, sealed-bid auction. Generally, sellers will choose the mechanism that guarantees them the most profit; this choice will depend on bidder risk attitudes and bidder beliefs about an object's value. If bidders are risk neutral and have independent valuation estimates, all auction types will yield the same outcome.

Decisions regarding how to auction a large number of objects, individually or as a group, and whether to disclose information are nontrivial. Sellers must also be wary of bidder collusion or fraud. Auctions now occur online using a variety of mechanisms and for the sale of a wide variety of goods. The primary strategic difference for bidders in such auctions arises due to the hard ending times imposed on many sites.

KEY TERMS

all-pay auction (643)
ascending auction (634)
common value (635)
descending auction (634)
Dutch auction (634)
English auction (634)
first-price auction (635)
objective value (635)
open outcry (634)
private value (636)
proxy bidding (654)
reservation price (654)
reserve price (645)
sealed bid (635)
second-price auction (635)
shading (640)

SOLVED EXERCISES

S1. A house painter has a regular contract to work for a builder. On these jobs, her cost estimates are generally right: sometimes a little high, sometimes a little low, but correct on average. When her regular work is slack, she bids competitively for other jobs. "Those are different," she says. "They almost always end up costing more than I estimate." If we assume that her estimating skills do not differ between the two types of jobs, what can explain the difference?

S2. Consider an auction where n identical objects are offered, and there are $(n + 1)$ bidders. The actual value of an object is the same for all bidders and equal for all objects, but each bidder gets only an independent estimate, subject to error, of this common value. The bidders submit sealed bids. The top n bidders get one object each, and each pays what she has bid. What considerations will affect your bidding strategy? How?

S3. You are in the market for a used car and see an ad for the model that you like. The owner has not set a price but invites potential buyers to make offers. Your prepurchase inspection gives you only a very rough idea of the value of the car; you think it is equally likely to be anywhere in the range of \$1,000 to \$5,000 (so your calculation of the average of this value is \$3,000). The current owner knows the exact value and will accept your offer if it exceeds that value. If your offer is accepted and you get the car, then you will find out the truth. But you have some special repair skills and know that when you own the car, you will be able to work on it and increase its value by a third (33.3 . . . %) of whatever it is worth.

(a) What is your expected profit if you offer \$3,000? Should you make such an offer?

(b) What is the highest offer that you can make without losing money on the deal?

S4. In this problem, we consider a special case of the first-price, sealed-bid auction and show what the equilibrium amount of bid shading should be. Consider a first-price, sealed-bid auction with n risk-neutral bidders. Each bidder has a private value independently drawn from a uniform distribution on [0,1]. That is, for each bidder, all values between 0 and 1 are equally likely. The complete strategy of each bidder is a "bid function" that will tell us, for any value v, what amount $b(v)$ that bidder will choose to bid. Deriving the equilibrium bid functions requires solving a differential equation, but instead of asking you to derive the equilibrium using a differential equation, this problem proposes a candidate equilibrium and asks you to confirm that it is indeed a Nash equilibrium.

It is proposed that the equilibrium-bid function for $n = 2$ is $b(v) = v/2$ for each of the two bidders. That is, if we have two bidders, each should bid half her value, which represents considerable shading.

(a) Suppose you're bidding against just one opponent whose value is uniformly distributed on [0, 1], and who always bids half her value. What is the probability that you will win if you bid $b = 0.1$? If you bid $b = 0.4$? If you bid $b = 0.6$?

(b) Put together the answers to part (a). What is the correct mathematical expression for Pr(win), the probability that you win, as a function of your bid b?

(c) Find an expression for the expected profit you make when your value is v and your bid is b, given that your opponent is bidding half her value. Remember that there are two cases: either you win the auction, or you lose the auction. You need to average the profit between these two cases.

(d) What is the value of b that maximizes your expected profit? This should be a function of your value v.

(e) Use your results to argue that it is a Nash equilibrium for both bidders to follow the same bid function $b(v) = v/2$.

S5. **(Optional)** This question looks at the equilibrium bidding strategies of all-pay auctions, in which bidders have private values for the good, as opposed to the discussion in Section 4, where the all-pay auction is for a good with a publicly known value. For the all-pay auction with private values distributed uniformly between 0 and 1, the Nash equilibrium bid function is $b(v) = [(n - 1)/n]v^n$.

(a) Plot graphs of $b(v)$ for the case $n = 2$ and for the case $n = 3$.

(b) Are the bids increasing in the number of bidders or decreasing in the number of bidders? Your answer might depend on n and v. That is, bids are sometimes increasing in n, and sometimes decreasing in n.

(c) Prove that the function given above is really the Nash-equilibrium bid function. Use a similar approach to that of Exercise S4. Remember that in an all-pay auction, you pay your bid even when you lose, so your payoff is $v - b$ when you win, and $-b$ when you lose.

UNSOLVED EXERCISES

U1. "In the presence of very risk-averse bidders, a person selling her house in an auction will have a high expected profit by using a first-price, sealed-bid auction." True or false? Explain your answer.

U2. Suppose that three risk-neutral bidders are interested in purchasing a Princess Beanie Baby. The bidders (numbered 1 through 3) have valuations of \$12, \$14, and \$16, respectively. The bidders will compete in auctions as described in parts (a) through (d); in each case, bids can be made in \$1 increments at any value from \$5 to \$25.

- **(a)** Which bidder wins an open-outcry English auction? What are the final price paid and the profit to the winning bidder?
- **(b)** Which bidder wins a second-price, sealed-bid auction? What are the final price paid and the profit to the winning bidder? Contrast your answer here with that for part (a). What is the cause of the difference in profits in these two cases?
- **(c)** In a sealed-bid, first-price auction, all the bidders will bid a positive amount (at least \$1) less than their true valuations. What is the likely outcome in this auction? Contrast your answer with those for parts (a) and (b). Does the seller of the Princess Beanie Baby have any clear reason to choose one of these auction mechanisms over the other?
- **(d)** Risk-averse bidders would reduce the shading of their bids in part (c); assume, for the purposes of this question, that they do not shade at all. If that were true, what would be the winning price (and profit for the bidder) in part (c)? Does the seller care about which type of auction she chooses? Why?

U3. You are a turnaround artist, specializing in identifying underperforming companies, buying them, improving their performance and stock price, and then selling them. You have found such a prospect, Sicco. This company's marketing department is mediocre; you believe that if you take over the company, you will increase its value by 75% of whatever it was before. But its accounting department is very good; it can conceal assets, liabilities, and transactions to a point where the company's true value is hard for outsiders to identify. (But insiders know the truth perfectly.) You think that the company's value in the hands of its current management is somewhere between \$10 million and \$110 million, uniformly distributed over this range. The current management will sell the company to you if, and only if, your bid exceeds the true value known to them.

- **(a)** If you bid \$110 million for the company, your bid will surely succeed. Is your expected profit positive?

(b) If you bid \$50 million for the company, what is the probability that your bid succeeds? What is your expected profit if you do succeed in buying the company? Therefore, at the point in time when you make your bid of \$50 million, what is your expected profit? (Warning: In calculating this expectation, don't forget the probability of your getting the company.)

(c) What should you bid if you want to maximize your expected profit? (Hint: Assume it is X million dollars. Carry out the same analysis as in part (b) above, and find an algebraic expression for your expected profit as seen from the point in time when you are making your bid. Then choose X to maximize this expression.)

U4. The idea of the winner's curse can be expressed slightly differently from its usage in the chapter: "The only time your bid matters is when you win, which happens when your estimate is higher than the estimates of all the other bidders. Therefore you should focus on this case. That is, you should always act as if all the others have received estimates lower than yours, and use this 'information' to revise your own estimate." Here we ask you to apply this idea to a very different situation.

A jury consists of 12 people who hear and see evidence presented at a trial and collectively reach a verdict of guilt or innocence. Simplifying the process somewhat, assume that the jurors hold a single simultaneous vote to determine the verdict. Each juror is asked to vote Guilty or Not guilty. The accused is convicted if all 12 vote Guilty and is acquitted if one or more vote Not guilty; this is known as the unanimity rule. Each juror's objective is to arrive at a verdict that is the most accurate verdict in light of the evidence, but each juror interprets the evidence in accord with her own thinking and experience. Thus, she arrives at an estimate of the guilt or the innocence of the accused that is individual and private.

(a) If jurors vote truthfully—that is, in accordance with their individual private estimates of the guilt of the accused—will the verdict be Not guilty more often under a unanimity rule or under a majority rule, where the accused is convicted if seven jurors vote Guilty? Explain. What might we call the "juror's curse" in this situation?

(b) Now consider the case in which each juror votes strategically, taking into account the potential problems of the juror's curse and using all the devices of information inference that we have studied. Are individual jurors more likely to vote Guilty under a unanimity rule when voting truthfully or strategically? Explain.

(c) Do you think strategic voting to account for the juror's curse would produce too many Guilty verdicts? Why or why not?

U5. **(Optional)** This exercise is a continuation of Exercise S4; it looks at the general case where n is any positive integer. It is proposed that the equilibrium-bid function with n bidders is $b(v) = v(n - 1)/n$. For $n = 2$, we have the case explored in Exercise S4: each of the bidders bids half of her value. If there are nine bidders ($n = 9$), then each should bid 9/10 of her value, and so on.

(a) Now there are $n - 1$ other bidders bidding against you, each using the bid function $b(v) = v(n - 1)/n$. For the moment, let's focus on just one of your rival bidders. What is the probability that she will submit a bid less than 0.1? Less than 0.4? Less than 0.6?

(b) Using the above results, find an expression for the probability that the other bidder has a bid less than your bid amount b.

(c) Recall that there are $n - 1$ other bidders, all using the same bid function. What is the probability that your bid b is larger than *all* of the other bids? That is, find an expression for Pr(win), the probability that you win, as a function of your bid b.

(d) Use this result to find an expression for your expected profit when your value is v and your bid is b.

(e) What is the value of b that maximizes your expected profit?

Use your results to argue that it is a Nash equilibrium for all n bidders to follow the same bid function $b(v) = v(n - 1)/n$.