6 | Convolutional Neural Networks

In several of our previous examples, we have already come up against image data, which consist of pixels arranged in a 2D grid. Depending on whether we are looking at a black and white or color image, we might have either one or multiple numerical values corresponding to each pixel location. Until now, we have dealt with this rich structure in the least satisfying possible way. We simply threw away this spatial structure by flattening each image into a 1D vector, and fed it into a fully-connected network. These networks are invariant to the order of their inputs. We will get qualitatively identical results out of a multilayer perceptron whether we preserve the original order of our features or if we permute the columns of our design matrix before learning the parameters. Ideally, we would find a way to leverage our prior knowledge that nearby pixels are more related to each other.

In this chapter, we introduce convolutional neural networks (CNNs), a powerful family of neural networks that were designed for precisely this purpose. CNN-based network *architecures* now dominate the field of computer vision to such an extent that hardly anyone these days would develop a commercial application or enter a competition related to image recognition, object detection, or semantic segmentation, without basing their approach on them.

Modern 'convnets', as they are often called owe their design to inspirations from biology, group theory, and a healthy dose of experimental tinkering. In addition to their strong predictive performance, convolutional neural networks tend to be computationally efficient, both because they tend to require fewer parameters than dense architectures and also because convolutions are easy to parallelize across GPU cores. As a result, researchers have sought to apply convnets whenever possible, and increasingly they have emerged as credible competitors even on tasks with 1D sequence structure, such as audio, text, and time series analysis, where recurrent neural networks (introduced in the next chapter) are conventionally used. Some clever adaptations of CNNs have also brought them to bear on graph-structured data and in recommender systems.

First, we will walk through the basic operations that comprise the backbone of all modern convolutional networks. These include the convolutional layers themselves, nitty-gritty details including padding and stride, the pooling layers used to aggregate information across adjacent spatial regions, the use of multiple *channels* (also called *filters*) at each layer, and a careful discussion of the structure of modern architectures. We will conclude the chapter with a full working example of LeNet, the first convolutional network successfully deployed, long before the rise of modern deep learning. In the next chapter we will dive into full implementations of some of the recent popular neural networks whose designs are representative of most of the techniques commonly used to design modern convolutional neural networks.

6.1 From Dense Layers to Convolutions

The models that we have discussed so far are fine options if you are dealing with *tabular* data. By *tabular* we mean that the data consists of rows corresponding to examples and columns corresponding to features. With tabular data, we might anticipate that pattern we seek could require modeling interactions among the features, but do not assume anything a priori about which features are related to each other or in what way.

Sometimes we truly may not have any knowledge to guide the construction of more cleverly-organized architectures. In these cases, a multilayer perceptron is often the best that we can do. However, once we start dealing with high-dimensional perceptual data, these *structure-less* networks can grow unwieldy.

For instance, let's return to our running example of distinguishing cats from dogs. Say that we do a thorough job in data collection, collecting an annotated sets of high-quality 1-megapixel photographs. This means that the input into a network has *1 million dimensions*. Even an aggressive reduction to *1,000 hidden dimensions* would require a *dense* (fully-connected) layer to support 10^9 parameters. Unless we have an extremely large dataset (perhaps billions?), lots of GPUs, a talent for extreme distributed optimization, and an extraordinary amount of patience, learning the parameters of this network may turn out to be impossible.

A careful reader might object to this argument on the basis that 1 megapixel resolution may not be necessary. However, while you could get away with 100,000 pixels, we grossly underestimated the number of hidden nodes that it typically takes to learn good hidden representations of images. Learning a binary classifier with so many parameters might seem to require that we collect an enormous dataset, perhaps comparable to the number of dogs and cats on the planet. And yet both humans and computers are able to distinguish cats from dogs quite well, seemingly contradicting these conclusions. That is because images exhibit rich structure that is typically exploited by humans and machine learning models alike.

6.1.1 Invariances

Imagine that you want to detect an object in an image. It seems reasonable that whatever method we use to recognize objects should not be overly concerned with the precise *location* of the object shouldn't in the image. Ideally we could learn a system that would somehow exploit this knowledge. Pigs usually do not fly and planes usually do not swim. Nonetheless, we could still recognize a flying pig were one to appear. This ideas is taken to an extreme in the children's game 'Where's Waldo', an example is shown in Fig. 6.1.1. The game consists of a number of chaotic scenes bursting with activity and Waldo shows up somewhere in each (typically lurking in some unlikely location). The reader's goal is to locate him. Despite his characteristic outfit, this can be surprisingly difficult, due to the large number of confounders.

Fig. 6.1.1: Image via Walker Books

Back to images, the intuitions we have been discussing could be made more concrete yielding a few key principles for building neural networks for computer vision:

1. Our vision systems should, in some sense, respond similarly to the same object regardless of where it appears in the image (translation invariance).
2. Our visions systems should, in some sense, focus on local regions, without regard for what else is happening in the image at greater distances (locality).

Let's see how this translates into mathematics.

6.1.2 Constraining the MLP

To start off let's consider what an MLP would look like with $h \times w$ images as inputs (represented as matrices in math, and as 2D arrays in code), and hidden representations similarly organized as $h \times w$ matrices / 2D arrays. Let $x[i, j]$ and $h[i, j]$ denote pixel location (i, j) in an image and hidden representation, respectively. Consequently, to have each of the hw hidden nodes receive input from each of the hw inputs, we would switch from using weight matrices (as we did previously in MLPs) to representing our parameters as four-dimensional weight tensors.

We could formally express this dense layer as follows:

$$h[i, j] = u[i, j] + \sum_{k,l} W[i, j, k, l] \cdot x[k, l] = u[i, j] + \sum_{a,b} V[i, j, a, b] \cdot x[i + a, j + b]. \tag{6.1.1}$$

The switch from W to V is entirely cosmetic (for now) since there is a one-to-one correspondence between coefficients in both tensors. We simply re-index the subscripts (k, l) such that $k = i + a$ and $l = j + b$. In other words, we set $V[i, j, a, b] = W[i, j, i + a, j + b]$. The indices a, b run over both positive and negative offsets, covering the entire image. For any given location (i, j) in the hidden layer $h[i, j]$, we compute its value by summing over pixels in x, centered around (i, j) and weighted by $V[i, j, a, b]$.

Now let's invoke the first principle we established above: *translation invariance*. This implies that a shift in the inputs x should simply lead to a shift in the activations h. This is only possible if V

and u do not actually depend on (i, j), i.e., we have $V[i, j, a, b] = V[a, b]$ and u is a constant. As a result we can simplify the definition for h.

$$h[i, j] = u + \sum_{a,b} V[a, b] \cdot x[i + a, j + b]. \tag{6.1.2}$$

This is a convolution! We are effectively weighting pixels $(i + a, j + b)$ in the vicinity of (i, j) with coefficients $V[a, b]$ to obtain the value $h[i, j]$. Note that $V[a, b]$ needs many fewer coefficients than $V[i, j, a, b]$. For a 1 megapixel image it has at most 1 million coefficients. This is 1 million fewer parameters since it no longer depends on the location within the image. We have made significant progress!

Now let's invoke the second principle—*locality*. As motivated above, we believe that we shouldn't have to look very far away from (i, j) in order to glean relevant information to assess what is going on at $h[i, j]$. This means that outside some range $|a|, |b| > \Delta$, we should set $V[a, b] = 0$. Equivalently, we can rewrite $h[i, j]$ as

$$h[i, j] = u + \sum_{a=-\Delta}^{\Delta} \sum_{b=-\Delta}^{\Delta} V[a, b] \cdot x[i + a, j + b]. \tag{6.1.3}$$

This, in a nutshell is the convolutional layer. When the local region (also called a *receptive field*) is small, the difference as compared to a fully-connected network can be dramatic. While previously, we might have required billions of parameters to represent just a single layer in an image-processing network, we now typically need just a few hundred. The price that we pay for this drastic modification is that our features will be translation invariant and that our layer can only take local information into account. All learning depends on imposing inductive bias. When that bias agrees with reality, we get sample-efficient models that generalize well to unseen data. But of course, if those biases do not agree with reality, e.g., if images turned out not to be translation invariant, our models may not generalize well.

6.1.3 Convolutions

Let's briefly review why the above operation is called a *convolution*. In mathematics, the convolution between two functions, say $f, g : \mathbb{R}^d \to R$ is defined as

$$[f \circledast g](x) = \int_{\mathbb{R}^d} f(z)g(x - z)dz. \tag{6.1.4}$$

That is, we measure the overlap between f and g when both functions are shifted by x and "flipped". Whenever we have discrete objects, the integral turns into a sum. For instance, for vectors defined on ℓ_2, i.e., the set of square summable infinite dimensional vectors with index running over $\mathbb{Z}$ we obtain the following definition.

$$[f \circledast g](i) = \sum_a f(a)g(i - a). \tag{6.1.5}$$

For two-dimensional arrays, we have a corresponding sum with indices (i, j) for f and $(i-a, j-b)$ for g respectively. This looks similar to definition above, with one major difference. Rather than using $(i+a, j+b)$, we are using the difference instead. Note, though, that this distinction is mostly cosmetic since we can always match the notation by using $\tilde{V}[a, b] = V[-a, -b]$ to obtain $h = x \circledast \tilde{V}$. Also note that the original definition is actually a *cross correlation*. We will come back to this in the following section.

6.1.4 Waldo Revisited

Let's see what this looks like if we want to build an improved Waldo detector. The convolutional layer picks windows of a given size and weighs intensities according to the mask V, as demonstrated in Fig. 6.1.2. We expect that wherever the "waldoness" is highest, we will also find a peak in the hidden layer activations.

Fig. 6.1.2: Find Waldo.

There is just a problem with this approach: so far we blissfully ignored that images consist of 3 channels: red, green and blue. In reality, images are quite two-dimensional objects but rather as a 3rd order tensor, e.g., with shape $1024 \times 1024 \times 3$ pixels. Only two of these axes concern spatial relationships, while the 3rd can be regarded as assigning a multidimensional representation *to each pixel location*.

We thus index **x** as $x[i, j, k]$. The convolutional mask has to adapt accordingly. Instead of $V[a, b]$ we now have $V[a, b, c]$.

Moreover, just as our input consists of a 3rd order tensor it turns out to be a good idea to similarly formulate our hidden representations as 3rd order tensors. In other words, rather than just having a 1D representation corresponding to each spatial location, we want to have a multidimensional hidden representations corresponding to each spatial location. We could think of the hidden representation as comprising a number of 2D grids stacked on top of each other. These are sometimes called *channels* or *feature maps*. Intuitively you might imagine that at lower layers, some channels specialize to recognizing edges, We can take care of this by adding a fourth coordinate to V via $V[a, b, c, d]$. Putting all together we have:

$$h[i, j, k] = \sum_{a=-\Delta}^{\Delta} \sum_{b=-\Delta}^{\Delta} \sum_{c} V[a, b, c, k] \cdot x[i + a, j + b, c]. \tag{6.1.6}$$

This is the definition of a convolutional neural network layer. There are still many operations that we need to address. For instance, we need to figure out how to combine all the activations to a single output (e.g., whether there is a Waldo in the image). We also need to decide how to compute things efficiently, how to combine multiple layers, and whether it is a good idea to have many narrow or a few wide layers. All of this will be addressed in the remainder of the chapter.

Summary

- Translation invariance in images implies that all patches of an image will be treated in the same manner.
- Locality means that only a small neighborhood of pixels will be used for computation.
- Channels on input and output allows for meaningful feature analysis.

Exercises

1. Assume that the size of the convolution mask is $\Delta = 0$. Show that in this case the convolutional mask implements an MLP independently for each set of channels.
2. Why might translation invariance not be a good idea after all? Does it make sense for pigs to fly?
3. What happens at the boundary of an image?
4. Derive an analogous convolutional layer for audio.
5. What goes wrong when you apply the above reasoning to text? Hint: what is the structure of language?
6. Prove that $f \circledast g = g \circledast f$.

6.2 Convolutions for Images

Now that we understand how convolutional layers work in theory, we are ready to see how this works in practice. Since we have motivated convolutional neural networks by their applicability to image data, we will stick with image data in our examples, and begin by revisiting the convolutional layer that we introduced in the previous section. We note that strictly speaking, *convolutional* layers are a slight misnomer, since the operations are typically expressed as cross correlations.

6.2.1 The Cross-Correlation Operator

In a convolutional layer, an input array and a correlation kernel array are combined to produce an output array through a cross-correlation operation. Let's see how this works for two dimensions. In Fig. 6.2.1, the input is a two-dimensional array with a height of 3 and width of 3. We mark the shape of the array as 3×3 or (3, 3). The height and width of the kernel array are both 2. Common names for this array in the deep learning research community include *kernel* and *filter*. The shape of the kernel window (also known as the convolution window) is given precisely by the height and width of the kernel (here it is 2×2).

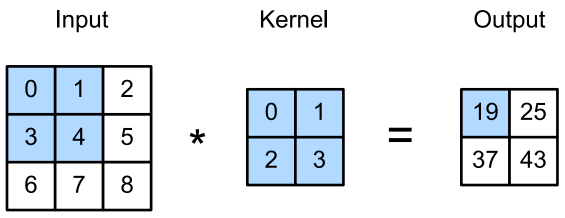

Fig. 6.2.1: Two-dimensional cross-correlation operation. The shaded portions are the first output element and the input and kernel array elements used in its computation: $0\times0+1\times1+3\times2+4\times3 = 19$.

In the two-dimensional cross-correlation operation, we begin with the convolution window positioned at the top-left corner of the input array and slide it across the input array, both from left to right and top to bottom. When the convolution window slides to a certain position, the input subarray contained in that window and the kernel array are multiplied (elementwise) and the resulting array is summed up yielding a single scalar value. This result if precisely the value of the output array at the corresponding location. Here, the output array has a height of 2 and width of 2 and the four elements are derived from the two-dimensional cross-correlation operation:

$$\begin{aligned} 0\times0+1\times1+3\times2+4\times3 &= 19,\\ 1\times0+2\times1+4\times2+5\times3 &= 25,\\ 3\times0+4\times1+6\times2+7\times3 &= 37,\\ 4\times0+5\times1+7\times2+8\times3 &= 43. \end{aligned} \tag{6.2.1}$$

Note that along each axis, the output is slightly *smaller* than the input. Because the kernel has a width greater than one, and we can only computer the cross-correlation for locations where the kernel fits wholly within the image, the output size is given by the input size $H \times W$ minus the size of the convolutional kernel $h \times w$ via $(H - h + 1) \times (W - w + 1)$. This is the case since we need enough space to 'shift' the convolutional kernel across the image (later we will see how to keep the size unchanged by padding the image with zeros around its boundary such that there is enough space to shift the kernel). Next, we implement the above process in the `corr2d` function. It accepts the input array `X` with the kernel array `K` and outputs the array `Y`.

```python
from mxnet import autograd, np, npx
from mxnet.gluon import nn
npx.set_np()

# Saved in the d2l package for later use
def corr2d(X, K):
    """Compute 2D cross-correlation."""
    h, w = K.shape
    Y = np.zeros((X.shape[0] - h + 1, X.shape[1] - w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            Y[i, j] = (X[i: i + h, j: j + w] * K).sum()
    return Y
```

We can construct the input array `X` and the kernel array `K` from the figure above to validate the output of the above implementations of the two-dimensional cross-correlation operation.

```
X = np.array([[0, 1, 2], [3, 4, 5], [6, 7, 8]])
K = np.array([[0, 1], [2, 3]])
corr2d(X, K)
```

```
array([[19., 25.],
       [37., 43.]])
```

6.2.2 Convolutional Layers

A convolutional layer cross-correlates the input and kernels and adds a scalar bias to produce an output. The parameters of the convolutional layer are precisely the values that constitute the kernel and the scalar bias. When training the models based on convolutional layers, we typically initialize the kernels randomly, just as we would with a fully-connected layer.

We are now ready to implement a two-dimensional convolutional layer based on the `corr2d` function defined above. In the `__init__` constructor function, we declare `weight` and `bias` as the two model parameters. The forward computation function `forward` calls the `corr2d` function and adds the bias. As with $h \times w$ cross-correlation we also refer to convolutional layers as $h \times w$ convolutions.

```
class Conv2D(nn.Block):
    def __init__(self, kernel_size, **kwargs):
        super(Conv2D, self).__init__(**kwargs)
        self.weight = self.params.get('weight', shape=kernel_size)
        self.bias = self.params.get('bias', shape=(1,))

    def forward(self, x):
        return corr2d(x, self.weight.data()) + self.bias.data()
```

6.2.3 Object Edge Detection in Images

Let's look at a simple application of a convolutional layer: detecting the edge of an object in an image by finding the location of the pixel change. First, we construct an 'image' of 6×8 pixels. The middle four columns are black (0) and the rest are white (1).

```
X = np.ones((6, 8))
X[:, 2:6] = 0
X
```

```
array([[1., 1., 0., 0., 0., 0., 1., 1.],
       [1., 1., 0., 0., 0., 0., 1., 1.],
       [1., 1., 0., 0., 0., 0., 1., 1.],
       [1., 1., 0., 0., 0., 0., 1., 1.],
       [1., 1., 0., 0., 0., 0., 1., 1.],
       [1., 1., 0., 0., 0., 0., 1., 1.]])
```

Next, we construct a kernel `K` with a height of 1 and width of 2. When we perform the cross-correlation operation with the input, if the horizontally adjacent elements are the same, the output is 0. Otherwise, the output is non-zero.

```
K = np.array([[1, -1]])
```

Enter X and our designed kernel K to perform the cross-correlation operations. As you can see, we will detect 1 for the edge from white to black and -1 for the edge from black to white. The rest of the outputs are 0.

```
Y = corr2d(X, K)
Y
```

```
array([[ 0.,  1.,  0.,  0.,  0., -1.,  0.],
       [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
       [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
       [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
       [ 0.,  1.,  0.,  0.,  0., -1.,  0.],
       [ 0.,  1.,  0.,  0.,  0., -1.,  0.]])
```

Let's apply the kernel to the transposed image. As expected, it vanishes. The kernel K only detects vertical edges.

```
corr2d(X.T, K)
```

```
array([[0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0.]])
```

6.2.4 Learning a Kernel

Designing an edge detector by finite differences [1, -1] is neat if we know this is precisely what we are looking for. However, as we look at larger kernels, and consider successive layers of convolutions, it might be impossible to specify precisely what each filter should be doing manually.

Now let's see whether we can learn the kernel that generated Y from X by looking at the (input, output) pairs only. We first construct a convolutional layer and initialize its kernel as a random array. Next, in each iteration, we will use the squared error to compare Y and the output of the convolutional layer, then calculate the gradient to update the weight. For the sake of simplicity, in this convolutional layer, we will ignore the bias.

We previously constructed the Conv2D class. However, since we used single-element assignments, Gluon has some trouble finding the gradient. Instead, we use the built-in Conv2D class provided by Gluon below.

```
# Construct a convolutional layer with 1 output channel
# (channels will be introduced in the following section)
# and a kernel array shape of (1, 2)
conv2d = nn.Conv2D(1, kernel_size=(1, 2))
```

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```
conv2d.initialize()

# The two-dimensional convolutional layer uses four-dimensional input and
# output in the format of (example channel, height, width), where the batch
# size (number of examples in the batch) and the number of channels are both 1
X = X.reshape(1, 1, 6, 8)
Y = Y.reshape(1, 1, 6, 7)

for i in range(10):
    with autograd.record():
        Y_hat = conv2d(X)
        l = (Y_hat - Y) ** 2
    l.backward()
    # For the sake of simplicity, we ignore the bias here
    conv2d.weight.data()[:] -= 3e-2 * conv2d.weight.grad()
    if (i + 1) % 2 == 0:
        print('batch %d, loss %.3f' % (i + 1, l.sum()))
```

```
batch 2, loss 4.949
batch 4, loss 0.831
batch 6, loss 0.140
batch 8, loss 0.024
batch 10, loss 0.004
```

As you can see, the error has dropped to a small value after 10 iterations. Now we will take a look at the kernel array we learned.

```
conv2d.weight.data().reshape(1, 2)
```

```
array([[ 0.9895    , -0.9873705]])
```

Indeed, the learned kernel array is remarkably close to the kernel array `K` we defined earlier.

6.2.5 Cross-Correlation and Convolution

Recall the observation from the previous section that cross-correlation and convolution are equivalent. In the figure above it is easy to see this correspondence. Simply flip the kernel from the bottom left to the top right. In this case the indexing in the sum is reverted, yet the same result can be obtained. In keeping with standard terminology with deep learning literature, we will continue to refer to the cross-correlation operation as a convolution even though, strictly-speaking, it is slightly different.

Summary

- The core computation of a two-dimensional convolutional layer is a two-dimensional cross-correlation operation. In its simplest form, this performs a cross-correlation operation on the two-dimensional input data and the kernel, and then adds a bias.
- We can design a kernel to detect edges in images.
- We can learn the kernel through data.

Exercises

1. Construct an image `X` with diagonal edges.
   - What happens if you apply the kernel `K` to it?
   - What happens if you transpose `X`?
   - What happens if you transpose `K`?
2. When you try to automatically find the gradient for the `Conv2D` class we created, what kind of error message do you see?
3. How do you represent a cross-correlation operation as a matrix multiplication by changing the input and kernel arrays?
4. Design some kernels manually.
   - What is the form of a kernel for the second derivative?
   - What is the kernel for the Laplace operator?
   - What is the kernel for an integral?
   - What is the minimum size of a kernel to obtain a derivative of degree d?

6.3 Padding and Stride

In the previous example, our input had a height and width of 3 and a convolution kernel with a height and width of 2, yielding an output with a height and a width of 2. In general, assuming the input shape is $n_h \times n_w$ and the convolution kernel window shape is $k_h \times k_w$, then the output shape will be

$$(n_h - k_h + 1) \times (n_w - k_w + 1). \tag{6.3.1}$$

Therefore, the output shape of the convolutional layer is determined by the shape of the input and the shape of the convolution kernel window.

In several cases we might want to incorporate particular techniques—padding and strides, regarding the size of the output:

- In general, since kernels generally have width and height greater than 1, that means that after applying many successive convolutions, we will wind up with an output that is much smaller than our input. If we start with a 240×240 pixel image, 10 layers of 5×5 convolutions reduce the image to 200×200 pixels, slicing off 30% of the image and with it obliterating any interesting information on the boundaries of the original image. *Padding* handles this issue.
- In some cases, we want to reduce the resolution drastically if say we find our
- original input resolution to be unwieldy. *Strides* can help in these instances.

6.3.1 Padding

As described above, one tricky issue when applying convolutional layers is that losing pixels on the perimeter of our image. Since we typically use small kernels, for any given convolution, we might only lose a few pixels, but this can add up as we apply many successive convolutional layers. One straightforward solution to this problem is to add extra pixels of filler around the boundary of our input image, thus increasing the effective size of the image Typically, we set the values of the extra pixels to 0. In Fig. 6.3.1, we pad a 3×5 input, increasing its size to 5×7. The corresponding output then increases to a 4×6 matrix.

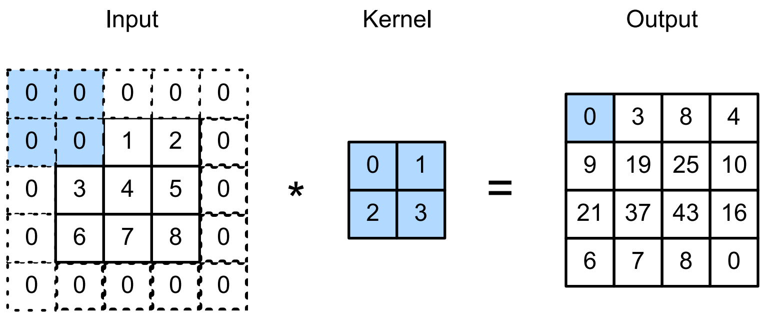

Fig. 6.3.1: Two-dimensional cross-correlation with padding. The shaded portions are the input and kernel array elements used by the first output element: $0 \times 0 + 0 \times 1 + 0 \times 2 + 0 \times 3 = 0$.

In general, if we add a total of p_h rows of padding (roughly half on top and half on bottom) and a total of p_w columns of padding (roughly half on the left and half on the right), the output shape will be

$$(n_h - k_h + p_h + 1) \times (n_w - k_w + p_w + 1). \tag{6.3.2}$$

This means that the height and width of the output will increase by p_h and p_w respectively.

In many cases, we will want to set $p_h = k_h - 1$ and $p_w = k_w - 1$ to give the input and output the same height and width. This will make it easier to predict the output shape of each layer when constructing the network. Assuming that k_h is odd here, we will pad $p_h/2$ rows on both sides of the height. If k_h is even, one possibility is to pad $\lceil p_h/2 \rceil$ rows on the top of the input and $\lfloor p_h/2 \rfloor$ rows on the bottom. We will pad both sides of the width in the same way.

Convolutional neural networks commonly use convolutional kernels with odd height and width values, such as 1, 3, 5, or 7. Choosing odd kernel sizes has the benefit that we can preserve the spatial dimensionality while padding with the same number of rows on top and bottom, and the same number of columns on left and right.

Moreover, this practice of using odd kernels and padding to precisely preserve dimensionality offers a clerical benefit. For any two-dimensional array `X`, when the kernels size is odd and the number of padding rows and columns on all sides are the same, producing an output with the same height and width as the input, we know that the output `Y[i, j]` is calculated by cross-correlation of the input and convolution kernel with the window centered on `X[i, j]`.

In the following example, we create a two-dimensional convolutional layer with a height and width of 3 and apply 1 pixel of padding on all sides. Given an input with a height and width of 8, we find that the height and width of the output is also 8.

```
from mxnet import np, npx
from mxnet.gluon import nn
npx.set_np()

# For convenience, we define a function to calculate the convolutional layer.
# This function initializes the convolutional layer weights and performs
# corresponding dimensionality elevations and reductions on the input and
# output
def comp_conv2d(conv2d, X):
    conv2d.initialize()
    # (1, 1) indicates that the batch size and the number of channels
    # (described in later chapters) are both 1
    X = X.reshape((1, 1) + X.shape)
    Y = conv2d(X)
    # Exclude the first two dimensions that do not interest us: batch and
    # channel
    return Y.reshape(Y.shape[2:])

# Note that here 1 row or column is padded on either side, so a total of 2
# rows or columns are added
conv2d = nn.Conv2D(1, kernel_size=3, padding=1)
X = np.random.uniform(size=(8, 8))
comp_conv2d(conv2d, X).shape
```

```
(8, 8)
```

When the height and width of the convolution kernel are different, we can make the output and input have the same height and width by setting different padding numbers for height and width.

```
# Here, we use a convolution kernel with a height of 5 and a width of 3. The
# padding numbers on both sides of the height and width are 2 and 1,
# respectively
conv2d = nn.Conv2D(1, kernel_size=(5, 3), padding=(2, 1))
comp_conv2d(conv2d, X).shape
```

```
(8, 8)
```

6.3.2 Stride

When computing the cross-correlation, we start with the convolution window at the top-left corner of the input array, and then slide it over all locations both down and to the right. In previous examples, we default to sliding one pixel at a time. However, sometimes, either for computational efficiency or because we wish to downsample, we move our window more than one pixel at a time, skipping the intermediate locations.

We refer to the number of rows and columns traversed per slide as the *stride*. So far, we have used strides of 1, both for height and width. Sometimes, we may want to use a larger stride. Fig. 6.3.2 shows a two-dimensional cross-correlation operation with a stride of 3 vertically and 2 horizontally. We can see that when the second element of the first column is output, the convolution window slides down three rows. The convolution window slides two columns to the right when the second element of the first row is output. When the convolution window slides three columns to the right on the input, there is no output because the input element cannot fill the window (unless we add another column of padding).

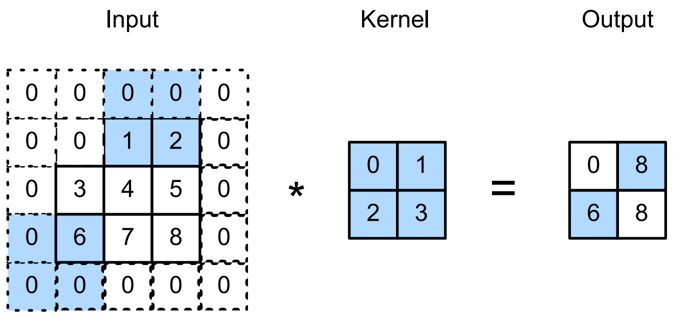

Fig. 6.3.2: Cross-correlation with strides of 3 and 2 for height and width respectively. The shaded portions are the output element and the input and core array elements used in its computation: $0\times0+0\times1+1\times2+2\times3=8$, $0\times0+6\times1+0\times2+0\times3=6$.

In general, when the stride for the height is s_h and the stride for the width is s_w, the output shape is

$$\lfloor(n_h-k_h+p_h+s_h)/s_h\rfloor \times \lfloor(n_w-k_w+p_w+s_w)/s_w\rfloor. \tag{6.3.3}$$

If we set $p_h=k_h-1$ and $p_w=k_w-1$, then the output shape will be simplified to $\lfloor(n_h+s_h-1)/s_h\rfloor \times \lfloor(n_w+s_w-1)/s_w\rfloor$. Going a step further, if the input height and width are divisible by the strides on the height and width, then the output shape will be $(n_h/s_h) \times (n_w/s_w)$.

Below, we set the strides on both the height and width to 2, thus halving the input height and width.

```
conv2d = nn.Conv2D(1, kernel_size=3, padding=1, strides=2)
comp_conv2d(conv2d, X).shape
```

```
(4, 4)
```

Next, we will look at a slightly more complicated example.

```
conv2d = nn.Conv2D(1, kernel_size=(3, 5), padding=(0, 1), strides=(3, 4))
comp_conv2d(conv2d, X).shape
```

```
(2, 2)
```

For the sake of brevity, when the padding number on both sides of the input height and width are p_h and p_w respectively, we call the padding (p_h, p_w). Specifically, when $p_h = p_w = p$, the padding is p. When the strides on the height and width are s_h and s_w, respectively, we call the stride (s_h, s_w). Specifically, when $s_h = s_w = s$, the stride is s. By default, the padding is 0 and the stride is 1. In practice we rarely use inhomogeneous strides or padding, i.e., we usually have $p_h = p_w$ and $s_h = s_w$.

Summary

- Padding can increase the height and width of the output. This is often used to give the output the same height and width as the input.
- The stride can reduce the resolution of the output, for example reducing the height and width of the output to only $1/n$ of the height and width of the input (n is an integer greater than 1).
- Padding and stride can be used to adjust the dimensionality of the data effectively.

Exercises

1. For the last example in this section, use the shape calculation formula to calculate the output shape to see if it is consistent with the experimental results.
2. Try other padding and stride combinations on the experiments in this section.
3. For audio signals, what does a stride of 2 correspond to?
4. What are the computational benefits of a stride larger than 1.

6.4 Multiple Input and Output Channels

While we have described the multiple channels that comprise each image (e.g., color images have the standard RGB channels to indicate the amount of red, green and blue), until now, we simplified all of our numerical examples by working with just a single input and a single output channel. This has allowed us to think of our inputs, convolutional kernels, and outputs each as two-dimensional arrays.

When we add channels into the mix, our inputs and hidden representations both become three-dimensional arrays. For example, each RGB input image has shape $3 \times h \times w$. We refer to this axis, with a size of 3, as the channel dimension. In this section, we will take a deeper look at convolution kernels with multiple input and multiple output channels.

6.4.1 Multiple Input Channels

When the input data contains multiple channels, we need to construct a convolution kernel with the same number of input channels as the input data, so that it can perform cross-correlation with the input data. Assuming that the number of channels for the input data is c_i, the number of input channels of the convolution kernel also needs to be c_i. If our convolution kernel's window shape is $k_h \times k_w$, then when $c_i = 1$, we can think of our convolution kernel as just a two-dimensional array of shape $k_h \times k_w$.

However, when $c_i > 1$, we need a kernel that contains an array of shape $k_h \times k_w$ *for each input channel*. Concatenating these c_i arrays together yields a convolution kernel of shape $c_i \times k_h \times k_w$. Since the input and convolution kernel each have c_i channels, we can perform a cross-correlation operation on the two-dimensional array of the input and the two-dimensional kernel array of the convolution kernel for each channel, adding the c_i results together (summing over the channels) to yield a two-dimensional array. This is the result of a two-dimensional cross-correlation between multi-channel input data and a *multi-input channel* convolution kernel.

In Fig. 6.4.1, we demonstrate an example of a two-dimensional cross-correlation with two input channels. The shaded portions are the first output element as well as the input and kernel array elements used in its computation: $(1\times1+2\times2+4\times3+5\times4)+(0\times0+1\times1+3\times2+4\times3) = 56$.

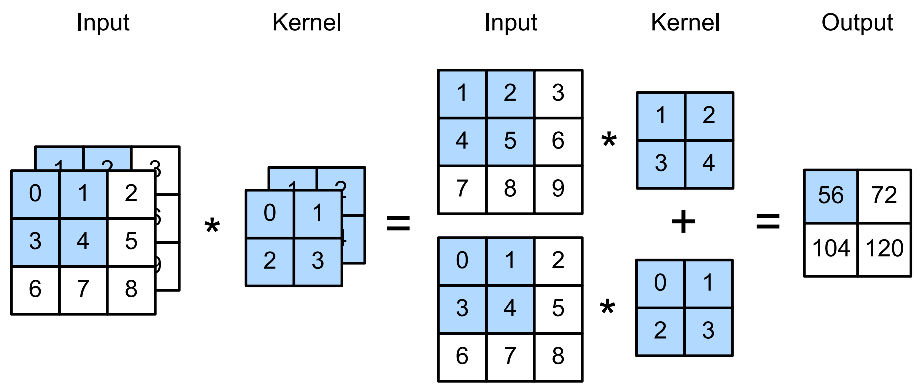

Fig. 6.4.1: Cross-correlation computation with 2 input channels. The shaded portions are the first output element as well as the input and kernel array elements used in its computation: $(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4) + (0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3) = 56$.

To make sure we really understand what is going on here, we can implement cross-correlation operations with multiple input channels ourselves. Notice that all we are doing is performing one cross-correlation operation per channel and then adding up the results using the `add_n` function.

```
import d2l
from mxnet import np, npx
npx.set_np()

def corr2d_multi_in(X, K):
    # First, traverse along the 0th dimension (channel dimension) of X and K.
    # Then, add them together by using * to turn the result list into a
    # positional argument of the add_n function
    return sum(d2l.corr2d(x, k) for x, k in zip(X, K))
```

We can construct the input array `X` and the kernel array `K` corresponding to the values in the above diagram to validate the output of the cross-correlation operation.

```
X = np.array([[[0, 1, 2], [3, 4, 5], [6, 7, 8]],
              [[1, 2, 3], [4, 5, 6], [7, 8, 9]]])
K = np.array([[[0, 1], [2, 3]], [[1, 2], [3, 4]]])

corr2d_multi_in(X, K)
```

```
array([[ 56.,  72.],
       [104., 120.]])
```

6.4.2 Multiple Output Channels

Regardless of the number of input channels, so far we always ended up with one output channel. However, as we discussed earlier, it turns out to be essential to have multiple channels at each layer. In the most popular neural network architectures, we actually increase the channel dimension as we go higher up in the neural network, typically downsampling to trade off spatial resolution for greater *channel depth*. Intuitively, you could think of each channel as responding to some different set of features. Reality is a bit more complicated than the most naive interpretations of this intuition since representations are not learned independent but are rather optimized to be jointly useful. So it may not be that a single channel learns an edge detector but rather that some direction in channel space corresponds to detecting edges.

Denote by c_i and c_o the number of input and output channels, respectively, and let k_h and k_w be the height and width of the kernel. To get an output with multiple channels, we can create a kernel array of shape $c_i \times k_h \times k_w$ for each output channel. We concatenate them on the output channel dimension, so that the shape of the convolution kernel is $c_o \times c_i \times k_h \times k_w$. In cross-correlation operations, the result on each output channel is calculated from the convolution kernel corresponding to that output channel and takes input from all channels in the input array.

We implement a cross-correlation function to calculate the output of multiple channels as shown below.

```
def corr2d_multi_in_out(X, K):
    # Traverse along the 0th dimension of K, and each time, perform
    # cross-correlation operations with input X. All of the results are merged
    # together using the stack function
    return np.stack([corr2d_multi_in(X, k) for k in K])
```

We construct a convolution kernel with 3 output channels by concatenating the kernel array `K` with `K+1` (plus one for each element in `K`) and `K+2`.

```
K = np.stack((K, K + 1, K + 2))
K.shape
```

```
(3, 2, 2, 2)
```

Below, we perform cross-correlation operations on the input array `X` with the kernel array `K`. Now the output contains 3 channels. The result of the first channel is consistent with the result of the previous input array `X` and the multi-input channel, single-output channel kernel.

```
corr2d_multi_in_out(X, K)
```

```
array([[[ 56.,  72.],
        [104., 120.]],

       [[ 76., 100.],
        [148., 172.]],

       [[ 96., 128.],
        [192., 224.]]])
```

6.4.3 1×1 Convolutional Layer

At first, a 1×1 convolution, i.e., $k_h = k_w = 1$, does not seem to make much sense. After all, a convolution correlates adjacent pixels. A 1×1 convolution obviously does not. Nonetheless, they are popular operations that are sometimes included in the designs of complex deep networks. Let's see in some detail what it actually does.

Because the minimum window is used, the 1×1 convolution loses the ability of larger convolutional layers to recognize patterns consisting of interactions among adjacent elements in the height and width dimensions. The only computation of the 1×1 convolution occurs on the channel dimension.

Fig. 6.4.2 shows the cross-correlation computation using the 1×1 convolution kernel with 3 input channels and 2 output channels. Note that the inputs and outputs have the same height and width. Each element in the output is derived from a linear combination of elements *at the same position* in the input image. You could think of the 1×1 convolutional layer as constituting a fully-connected layer applied at every single pixel location to transform the c_i corresponding input values into c_o output values. Because this is still a convolutional layer, the weights are tied across pixel location Thus the 1×1 convolutional layer requires $c_o \times c_i$ weights (plus the bias terms).

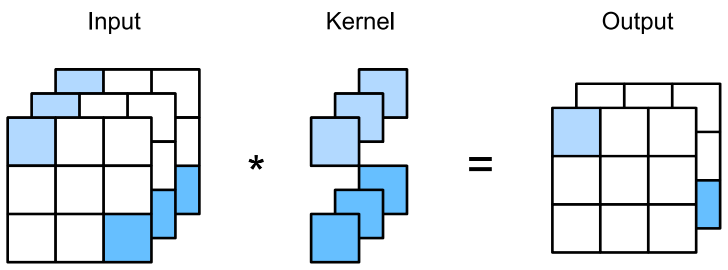

Fig. 6.4.2: The cross-correlation computation uses the 1×1 convolution kernel with 3 input channels and 2 output channels. The inputs and outputs have the same height and width.

Let's check whether this works in practice: we implement the 1×1 convolution using a fully-connected layer. The only thing is that we need to make some adjustments to the data shape before and after the matrix multiplication.

```
def corr2d_multi_in_out_1x1(X, K):
    c_i, h, w = X.shape
    c_o = K.shape[0]
```

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```
    X = X.reshape(c_i, h * w)
    K = K.reshape(c_o, c_i)
    Y = np.dot(K, X)  # Matrix multiplication in the fully connected layer
    return Y.reshape(c_o, h, w)
```

When performing 1×1 convolution, the above function is equivalent to the previously implemented cross-correlation function corr2d_multi_in_out. Let's check this with some reference data.

```
X = np.random.uniform(size=(3, 3, 3))
K = np.random.uniform(size=(2, 3, 1, 1))

Y1 = corr2d_multi_in_out_1x1(X, K)
Y2 = corr2d_multi_in_out(X, K)

np.abs(Y1 - Y2).sum() < 1e-6
```

```
array(True)
```

Summary

- Multiple channels can be used to extend the model parameters of the convolutional layer.
- The 1×1 convolutional layer is equivalent to the fully-connected layer, when applied on a per pixel basis.
- The 1×1 convolutional layer is typically used to adjust the number of channels between network layers and to control model complexity.

Exercises

1. Assume that we have two convolutional kernels of size k_1 and k_2 respectively (with no nonlinearity in between).
   - Prove that the result of the operation can be expressed by a single convolution.
   - What is the dimensionality of the equivalent single convolution?
   - Is the converse true?
2. Assume an input shape of $c_i \times h \times w$ and a convolution kernel with the shape $c_o \times c_i \times k_h \times k_w$, padding of (p_h, p_w), and stride of (s_h, s_w).
   - What is the computational cost (multiplications and additions) for the forward computation?
   - What is the memory footprint?
   - What is the memory footprint for the backward computation?
   - What is the computational cost for the backward computation?
3. By what factor does the number of calculations increase if we double the number of input channels c_i and the number of output channels c_o? What happens if we double the padding?

4. If the height and width of the convolution kernel is $k_h = k_w = 1$, what is the complexity of the forward computation?
5. Are the variables `Y1` and `Y2` in the last example of this section exactly the same? Why?
6. How would you implement convolutions using matrix multiplication when the convolution window is not 1×1?

6.5 Pooling

Often, as we process images, we want to gradually reduce the spatial resolution of our hidden representations, aggregating information so that the higher up we go in the network, the larger the receptive field (in the input) to which each hidden node is sensitive.

Often our ultimate task asks some global question about the image, e.g., *does it contain a cat?* So typically the nodes of our final layer should be sensitive to the entire input. By gradually aggregating information, yielding coarser and coarser maps, we accomplish this goal of ultimately learning a global representation, while keeping all of the advantages of convolutional layers at the intermediate layers of processing.

Moreover, when detecting lower-level features, such as edges (as discussed in Section 6.2), we often want our representations to be somewhat invariant to translation. For instance, if we take the image `X` with a sharp delineation between black and white and shift the whole image by one pixel to the right, i.e., `Z[i, j] = X[i, j+1]`, then the output for the new image `Z` might be vastly different. The edge will have shifted by one pixel and with it all the activations. In reality, objects hardly ever occur exactly at the same place. In fact, even with a tripod and a stationary object, vibration of the camera due to the movement of the shutter might shift everything by a pixel or so (high-end cameras are loaded with special features to address this problem).

This section introduces pooling layers, which serve the dual purposes of mitigating the sensitivity of convolutional layers to location and of spatially downsampling representations.

6.5.1 Maximum Pooling and Average Pooling

Like convolutional layers, pooling operators consist of a fixed-shape window that is slid over all regions in the input according to its stride, computing a single output for each location traversed by the fixed-shape window (sometimes known as the *pooling window*). However, unlike the cross-correlation computation of the inputs and kernels in the convolutional layer, the pooling layer contains no parameters (there is no *filter*). Instead, pooling operators are deterministic, typically calculating either the maximum or the average value of the elements in the pooling window. These operations are called *maximum pooling* (*max pooling* for short) and *average pooling*, respectively.

In both cases, as with the cross-correlation operator, we can think of the pooling window as starting from the top left of the input array and sliding across the input array from left to right and top to bottom. At each location that the pooling window hits, it computes the maximum or average value of the input subarray in the window (depending on whether *max* or *average* pooling is employed).

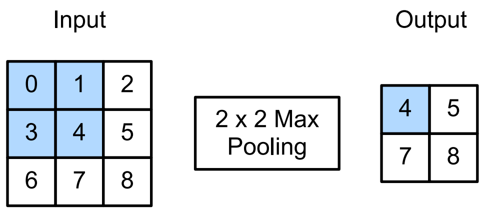


Fig. 6.5.1: Maximum pooling with a pooling window shape of 2×2. The shaded portions represent the first output element and the input element used for its computation: $\max(0, 1, 3, 4) = 4$

The output array in Fig. 6.5.1 above has a height of 2 and a width of 2. The four elements are derived from the maximum value of max:

$$\begin{aligned}\max(0, 1, 3, 4) &= 4,\\ \max(1, 2, 4, 5) &= 5,\\ \max(3, 4, 6, 7) &= 7,\\ \max(4, 5, 7, 8) &= 8.\end{aligned} \tag{6.5.1}$$

A pooling layer with a pooling window shape of $p \times q$ is called a $p \times q$ pooling layer. The pooling operation is called $p \times q$ pooling.

Let's return to the object edge detection example mentioned at the beginning of this section. Now we will use the output of the convolutional layer as the input for 2×2 maximum pooling. Set the convolutional layer input as `X` and the pooling layer output as `Y`. Whether or not the values of `X[i, j]` and `X[i, j+1]` are different, or `X[i, j+1]` and `X[i, j+2]` are different, the pooling layer outputs all include `Y[i, j]=1`. That is to say, using the 2×2 maximum pooling layer, we can still detect if the pattern recognized by the convolutional layer moves no more than one element in height and width.

In the code below, we implement the forward computation of the pooling layer in the `pool2d` function. This function is similar to the `corr2d` function in Section 6.2. However, here we have no kernel, computing the output as either the max or the average of each region in the input..

```
from mxnet import np, npx
from mxnet.gluon import nn
npx.set_np()

def pool2d(X, pool_size, mode='max'):
    p_h, p_w = pool_size
    Y = np.zeros((X.shape[0] - p_h + 1, X.shape[1] - p_w + 1))
    for i in range(Y.shape[0]):
        for j in range(Y.shape[1]):
            if mode == 'max':
                Y[i, j] = np.max(X[i: i + p_h, j: j + p_w])
            elif mode == 'avg':
                Y[i, j] = X[i: i + p_h, j: j + p_w].mean()
    return Y
```

We can construct the input array `X` in the above diagram to validate the output of the two-dimensional maximum pooling layer.

```
X = np.array([[0, 1, 2], [3, 4, 5], [6, 7, 8]])
pool2d(X, (2, 2))
```

```
array([[4., 5.],
       [7., 8.]])
```

At the same time, we experiment with the average pooling layer.

```
pool2d(X, (2, 2), 'avg')
```

```
array([[2., 3.],
       [5., 6.]])
```

6.5.2 Padding and Stride

As with convolutional layers, pooling layers can also change the output shape. And as before, we can alter the operation to achieve a desired output shape by padding the input and adjusting the stride. We can demonstrate the use of padding and strides in pooling layers via the two-dimensional maximum pooling layer MaxPool2D shipped in MXNet Gluon's `nn` module. We first construct an input data of shape `(1, 1, 4, 4)`, where the first two dimensions are batch and channel.

```
X = np.arange(16).reshape(1, 1, 4, 4)
X
```

```
array([[[[ 0.,  1.,  2.,  3.],
         [ 4.,  5.,  6.,  7.],
         [ 8.,  9., 10., 11.],
         [12., 13., 14., 15.]]]])
```

By default, the stride in the `MaxPool2D` class has the same shape as the pooling window. Below, we use a pooling window of shape `(3, 3)`, so we get a stride shape of `(3, 3)` by default.

```
pool2d = nn.MaxPool2D(3)
# Because there are no model parameters in the pooling layer, we do not need
# to call the parameter initialization function
pool2d(X)
```

```
array([[[[10.]]]])
```

The stride and padding can be manually specified.

```
pool2d = nn.MaxPool2D(3, padding=1, strides=2)
pool2d(X)
```

```
array([[[[ 5.,  7.],
         [13., 15.]]]])
```

Of course, we can specify an arbitrary rectangular pooling window and specify the padding and stride for height and width, respectively.

```
pool2d = nn.MaxPool2D((2, 3), padding=(1, 2), strides=(2, 3))
pool2d(X)
```

```
array([[[[ 0.,  3.],
         [ 8., 11.],
         [12., 15.]]]])
```

6.5.3 Multiple Channels

When processing multi-channel input data, the pooling layer pools each input channel separately, rather than adding the inputs of each channel by channel as in a convolutional layer. This means that the number of output channels for the pooling layer is the same as the number of input channels. Below, we will concatenate arrays `X` and `X+1` on the channel dimension to construct an input with 2 channels.

```
X = np.concatenate((X, X + 1), axis=1)
X
```

```
array([[[[ 0.,  1.,  2.,  3.],
         [ 4.,  5.,  6.,  7.],
         [ 8.,  9., 10., 11.],
         [12., 13., 14., 15.]],

        [[ 1.,  2.,  3.,  4.],
         [ 5.,  6.,  7.,  8.],
         [ 9., 10., 11., 12.],
         [13., 14., 15., 16.]]]])
```

As we can see, the number of output channels is still 2 after pooling.

```
pool2d = nn.MaxPool2D(3, padding=1, strides=2)
pool2d(X)
```

```
array([[[[ 5.,  7.],
         [13., 15.]],

        [[ 6.,  8.],
         [14., 16.]]]])
```

Summary

- Taking the input elements in the pooling window, the maximum pooling operation assigns the maximum value as the output and the average pooling operation assigns the average value as the output.
- One of the major functions of a pooling layer is to alleviate the excessive sensitivity of the convolutional layer to location.
- We can specify the padding and stride for the pooling layer.

- Maximum pooling, combined with a stride larger than 1 can be used to reduce the resolution.
- The pooling layer's number of output channels is the same as the number of input channels.

Exercises

1. Can you implement average pooling as a special case of a convolution layer? If so, do it.
2. Can you implement max pooling as a special case of a convolution layer? If so, do it.
3. What is the computational cost of the pooling layer? Assume that the input to the pooling layer is of size $c \times h \times w$, the pooling window has a shape of $p_h \times p_w$ with a padding of (p_h, p_w) and a stride of (s_h, s_w).
4. Why do you expect maximum pooling and average pooling to work differently?
5. Do we need a separate minimum pooling layer? Can you replace it with another operation?
6. Is there another operation between average and maximum pooling that you could consider (hint: recall the softmax)? Why might it not be so popular?

6.6 Convolutional Neural Networks (LeNet)

We are now ready to put all of the tools together to deploy your first fully-functional convolutional neural network. In our first encounter with image data we applied a multilayer perceptron (Section 4.2) to pictures of clothing in the Fashion-MNIST dataset. Each image in Fashion-MNIST consisted of a two-dimensional 28×28 matrix. To make this data amenable to multilayer perceptrons which anticipate receiving inputs as one-dimensional fixed-length vectors, we first flattened each image, yielding vectors of length 784, before processing them with a series of fully-connected layers.

Now that we have introduced convolutional layers, we can keep the image in its original spatially-organized grid, processing it with a series of successive convolutional layers. Moreover, because we are using convolutional layers, we can enjoy a considerable savings in the number of parameters required.

In this section, we will introduce one of the first published convolutional neural networks whose benefit was first demonstrated by Yann Lecun, then a researcher at AT&T Bell Labs, for the purpose of recognizing handwritten digits in images—LeNet5[103]. In the 90s, their experiments with LeNet gave the first compelling evidence that it was possible to train convolutional neural networks by backpropagation. Their model achieved outstanding results at the time (only matched by Support Vector Machines at the time) and was adopted to recognize digits for processing deposits in ATM machines. Some ATMs still run the code that Yann and his colleague Leon Bottou wrote in the 1990s!

[103] http://yann.lecun.com/exdb/lenet/

6.6.1 LeNet

In a rough sense, we can think LeNet as consisting of two parts: (i) a block of convolutional layers; and (ii) a block of fully-connected layers. Before getting into the weeds, let's briefly review the model in Fig. 6.6.1.

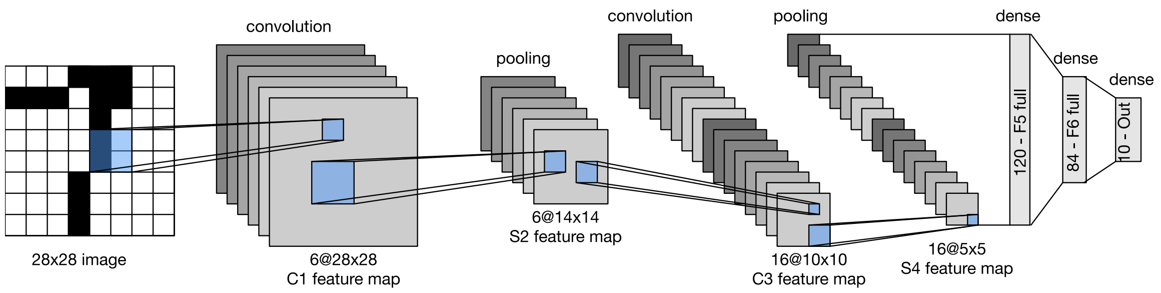


Fig. 6.6.1: Data flow in LeNet 5. The input is a handwritten digit, the output a probabilitiy over 10 possible outcomes.

The basic units in the convolutional block are a convolutional layer and a subsequent average pooling layer (note that max-pooling works better, but it had not been invented in the 90s yet). The convolutional layer is used to recognize the spatial patterns in the image, such as lines and the parts of objects, and the subsequent average pooling layer is used to reduce the dimensionality. The convolutional layer block is composed of repeated stacks of these two basic units. Each convolutional layer uses a 5×5 kernel and processes each output with a sigmoid activation function (again, note that ReLUs are now known to work more reliably, but had not been invented yet). The first convolutional layer has 6 output channels, and second convolutional layer increases channel depth further to 16.

However, coinciding with this increase in the number of channels, the height and width are shrunk considerably. Therefore, increasing the number of output channels makes the parameter sizes of the two convolutional layers similar. The two average pooling layers are of size 2×2 and take stride 2 (note that this means they are non-overlapping). In other words, the pooling layer downsamples the representation to be precisely *one quarter* the pre-pooling size.

The convolutional block emits an output with size given by (batch size, channel, height, width). Before we can pass the convolutional block's output to the fully-connected block, we must flatten each example in the minibatch. In other words, we take this 4D input and transform it into the 2D input expected by fully-connected layers: as a reminder, the first dimension indexes the examples in the minibatch and the second gives the flat vector representation of each example. LeNet's fully-connected layer block has three fully-connected layers, with 120, 84, and 10 outputs, respectively. Because we are still performing classification, the 10 dimensional output layer corresponds to the number of possible output classes.

While getting to the point where you truly understand what is going on inside LeNet may have taken a bit of work, you can see below that implementing it in a modern deep learning library is remarkably simple. Again, we will rely on the Sequential class.

```
import d2l
from mxnet import autograd, gluon, init, np, npx
from mxnet.gluon import nn
npx.set_np()

net = nn.Sequential()
net.add(nn.Conv2D(channels=6, kernel_size=5, padding=2, activation='sigmoid'),
        nn.AvgPool2D(pool_size=2, strides=2),
        nn.Conv2D(channels=16, kernel_size=5, activation='sigmoid'),
        nn.AvgPool2D(pool_size=2, strides=2),
        # Dense will transform the input of the shape (batch size, channel,
        # height, width) into the input of the shape (batch size,
        # channel * height * width) automatically by default
        nn.Dense(120, activation='sigmoid'),
        nn.Dense(84, activation='sigmoid'),
        nn.Dense(10))
```

As compared to the original network, we took the liberty of replacing the Gaussian activation in the last layer by a regular dense layer, which tends to be significantly more convenient to train. Other than that, this network matches the historical definition of LeNet5.

Next, let's take a look of an example. As shown in Fig. 6.6.2, we feed a single-channel example of size 28×28 into the network and perform a forward computation layer by layer printing the output shape at each layer to make sure we understand what is happening here.

```
X = np.random.uniform(size=(1, 1, 28, 28))
net.initialize()
for layer in net:
    X = layer(X)
    print(layer.name, 'output shape:\t', X.shape)
```

```
conv0 output shape:  (1, 6, 28, 28)
pool0 output shape:  (1, 6, 14, 14)
conv1 output shape:  (1, 16, 10, 10)
pool1 output shape:  (1, 16, 5, 5)
dense0 output shape:         (1, 120)
dense1 output shape:         (1, 84)
dense2 output shape:         (1, 10)
```

Note that the height and width of the representation at each layer throughout the convolutional block is reduced (compared to the previous layer). The convolutional layer uses a kernel with a height and width of 5, which with only 2 pixels of padding in the first convolutional layer and none in the second convolutional layer leads to reductions in both height and width by 2 and 4 pixels, respectively. Moreover each pooling layer halves the height and width. However, as we go up the stack of layers, the number of channels increases layer-over-layer from 1 in the input to 6 after the first convolutional layer and 16 after the second layer. Then, the fully-connected layer reduces dimensionality layer by layer, until emitting an output that matches the number of image classes.

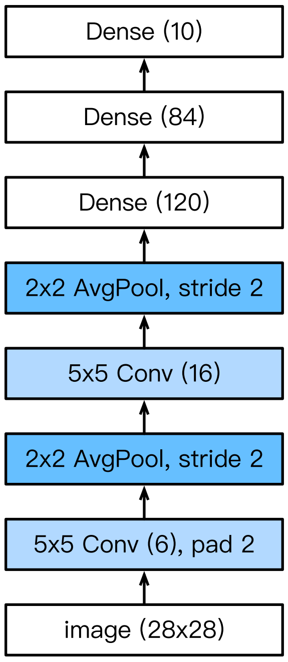


Fig. 6.6.2: Compressed notation for LeNet5

6.6.2 Data Acquisition and Training

Now that we have implemented the model, we might as well run some experiments to see what we can accomplish with the LeNet model. While it might serve nostalgia to train LeNet on the original MNIST dataset, that dataset has become too easy, with MLPs getting over 98% accuracy, so it would be hard to see the benefits of convolutional networks. Thus we will stick with Fashion-MNIST as our dataset because while it has the same shape (28×28 images), this dataset is notably more challenging.

```
batch_size = 256
train_iter, test_iter = d2l.load_data_fashion_mnist(batch_size=batch_size)
```

While convolutional networks may have few parameters, they can still be significantly more expensive to compute than a similarly deep multilayer perceptron so if you have access to a GPU, this might be a good time to put it into action to speed up training.

For evaluation, we need to make a slight modification to the `evaluate_accuracy` function that we described in Section 3.6. Since the full dataset lives on the CPU, we need to copy it to the GPU before we can compute our models. This is accomplished via the `as_in_context` function described in Section 5.6.

```
# Saved in the d2l package for later use
def evaluate_accuracy_gpu(net, data_iter, ctx=None):
    if not ctx:  # Query the first device the first parameter is on
        ctx = list(net.collect_params().values())[0].list_ctx()[0]
    metric = d2l.Accumulator(2)  # num_corrected_examples, num_examples
    for X, y in data_iter:
        X, y = X.as_in_context(ctx), y.as_in_context(ctx)
        metric.add(d2l.accuracy(net(X), y), y.size)
    return metric[0]/metric[1]
```

We also need to update our training function to deal with GPUs. Unlike the `train_epoch_ch3` defined in Section 3.6, we now need to move each batch of data to our designated context (hopefully, the GPU) prior to making the forward and backward passes.

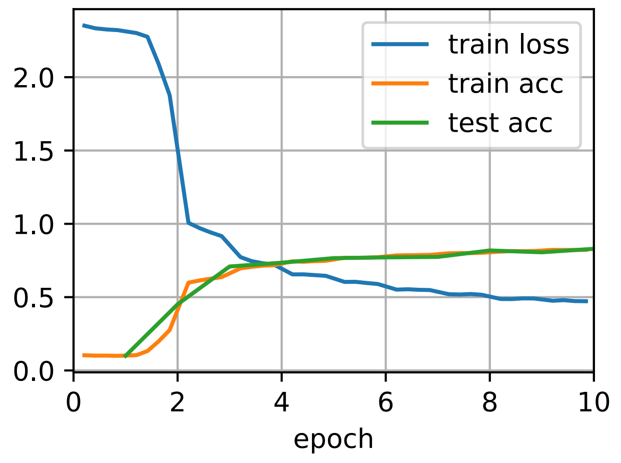
The training function `train_ch5` is also very similar to `train_ch3` defined in Section 3.6. Since we will deal with networks with tens of layers now, the function will only support Gluon models. We initialize the model parameters on the device indicated by `ctx`, this time using the Xavier initializer. The loss function and the training algorithm still use the cross-entropy loss function and minibatch stochastic gradient descent. Since each epoch takes tens of seconds to run, we visualize the training loss in a finer granularity.

```
# Saved in the d2l package for later use
def train_ch5(net, train_iter, test_iter, num_epochs, lr, ctx=d2l.try_gpu()):
    net.initialize(force_reinit=True, ctx=ctx, init=init.Xavier())
    loss = gluon.loss.SoftmaxCrossEntropyLoss()
    trainer = gluon.Trainer(net.collect_params(),
                            'sgd', {'learning_rate': lr})
    animator = d2l.Animator(xlabel='epoch', xlim=[0, num_epochs],
                            legend=['train loss', 'train acc', 'test acc'])
    timer = d2l.Timer()
    for epoch in range(num_epochs):
        metric = d2l.Accumulator(3)  # train_loss, train_acc, num_examples
        for i, (X, y) in enumerate(train_iter):
            timer.start()
            # Here is the only difference compared to train_epoch_ch3
            X, y = X.as_in_context(ctx), y.as_in_context(ctx)
            with autograd.record():
                y_hat = net(X)
                l = loss(y_hat, y)
            l.backward()
            trainer.step(X.shape[0])
            metric.add(l.sum(), d2l.accuracy(y_hat, y), X.shape[0])
            timer.stop()
            train_loss, train_acc = metric[0]/metric[2], metric[1]/metric[2]
            if (i+1) % 50 == 0:
                animator.add(epoch + i/len(train_iter),
                             (train_loss, train_acc, None))
        test_acc = evaluate_accuracy_gpu(net, test_iter)
        animator.add(epoch+1, (None, None, test_acc))
    print('loss %.3f, train acc %.3f, test acc %.3f' % (
        train_loss, train_acc, test_acc))
    print('%.1f exampes/sec on %s' % (metric[2]*num_epochs/timer.sum(), ctx))
```

Now let's train the model.

```
lr, num_epochs = 0.9, 10
train_ch5(net, train_iter, test_iter, num_epochs, lr)
```

```
loss 0.472, train acc 0.823, test acc 0.830
54537.5 exampes/sec on gpu(0)
```



Summary

- A convolutional neural network (in short, ConvNet) is a network using convolutional layers.
- In a ConvNet we alternate between convolutions, nonlinearities and often also pooling operations.
- Ultimately the resolution is reduced prior to emitting an output via one (or more) dense layers.
- LeNet was the first successful deployment of such a network.

Exercises

1. Replace the average pooling with max pooling. What happens?
2. Try to construct a more complex network based on LeNet to improve its accuracy.
    - Adjust the convolution window size.
    - Adjust the number of output channels.
    - Adjust the activation function (ReLU?).
    - Adjust the number of convolution layers.
    - Adjust the number of fully connected layers.
    - Adjust the learning rates and other training details (initialization, epochs, etc.)
3. Try out the improved network on the original MNIST dataset.
4. Display the activations of the first and second layer of LeNet for different inputs (e.g., sweaters, coats).