- $\mathrm{Cov}\left[aX, bY\right] = ab\mathrm{Cov}\left[X, Y\right]$
- $\mathrm{Cov}\left[X+a, Y+b\right] = \mathrm{Cov}\left[X, Y\right]$
- $\mathrm{Cov}\left[\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right] = \sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{Cov}\left[X_i, Y_j\right]$

Correlation

$$\rho\left[X, Y\right] = \frac{\mathrm{Cov}\left[X, Y\right]}{\sqrt{\mathbb{V}\left[X\right]\mathbb{V}\left[Y\right]}}$$

Independence

$$X \perp\!\!\!\perp Y \implies \rho\left[X, Y\right] = 0 \iff \mathrm{Cov}\left[X, Y\right] = 0 \iff \mathbb{E}\left[XY\right] = \mathbb{E}\left[X\right]\mathbb{E}\left[Y\right]$$

Sample variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$$

Conditional variance

- $\mathbb{V}\left[Y \mid X\right] = \mathbb{E}\left[(Y - \mathbb{E}\left[Y \mid X\right])^2 \mid X\right] = \mathbb{E}\left[Y^2 \mid X\right] - \mathbb{E}\left[Y \mid X\right]^2$
- $\mathbb{V}\left[Y\right] = \mathbb{E}\left[\mathbb{V}\left[Y \mid X\right]\right] + \mathbb{V}\left[\mathbb{E}\left[Y \mid X\right]\right]$

6 Inequalities

Cauchy-Schwarz

$$\mathbb{E}\left[XY\right]^2 \le \mathbb{E}\left[X^2\right]\mathbb{E}\left[Y^2\right]$$

Markov

$$\mathbb{P}\left[\varphi(X) \ge t\right] \le \frac{\mathbb{E}\left[\varphi(X)\right]}{t}$$

Chebyshev

$$\mathbb{P}\left[|X - \mathbb{E}\left[X\right]| \ge t\right] \le \frac{\mathbb{V}\left[X\right]}{t^2}$$

Chernoff

$$\mathbb{P}\left[X \ge (1+\delta)\mu\right] \le \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right) \quad \delta > -1$$

Hoeffding

$$X_1, \ldots, X_n \text{ independent } \wedge\ \mathbb{P}\left[X_i \in [a_i, b_i]\right] = 1\ \wedge\ 1 \le i \le n$$

$$\mathbb{P}\left[\bar{X} - \mathbb{E}\left[\bar{X}\right] \ge t\right] \le e^{-2nt^2} \quad t > 0$$

$$\mathbb{P}\left[|\bar{X} - \mathbb{E}\left[\bar{X}\right]| \ge t\right] \le 2\exp\left\{-\frac{2n^2t^2}{\sum_{i=1}^{n}(b_i - a_i)^2}\right\} \quad t > 0$$

Jensen

$$\mathbb{E}\left[\varphi(X)\right] \ge \varphi(\mathbb{E}\left[X\right]) \quad \varphi \text{ convex}$$

7 Distribution Relationships

Binomial

- $X_i \sim \mathrm{Bern}\left(p\right) \implies \sum_{i=1}^{n} X_i \sim \mathrm{Bin}\left(n, p\right)$
- $X \sim \mathrm{Bin}\left(n, p\right), Y \sim \mathrm{Bin}\left(m, p\right) \implies X + Y \sim \mathrm{Bin}\left(n+m, p\right)$
- $\lim_{n\to\infty} \mathrm{Bin}\left(n, p\right) = \mathrm{Po}\left(np\right)$ (n large, p small)
- $\lim_{n\to\infty} \mathrm{Bin}\left(n, p\right) = \mathcal{N}\left(np, np(1-p)\right)$ (n large, p far from 0 and 1)

Negative Binomial

- $X \sim \mathrm{NBin}\left(1, p\right) = \mathrm{Geo}\left(p\right)$
- $X \sim \mathrm{NBin}\left(r, p\right) = \sum_{i=1}^{r} \mathrm{Geo}\left(p\right)$
- $X_i \sim \mathrm{NBin}\left(r_i, p\right) \implies \sum X_i \sim \mathrm{NBin}\left(\sum r_i, p\right)$
- $X \sim \mathrm{NBin}\left(r, p\right).\ Y \sim \mathrm{Bin}\left(s+r, p\right) \implies \mathbb{P}\left[X \le s\right] = \mathbb{P}\left[Y \ge r\right]$

Poisson

- $X_i \sim \mathrm{Po}\left(\lambda_i\right) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_{i=1}^{n} X_i \sim \mathrm{Po}\left(\sum_{i=1}^{n} \lambda_i\right)$
- $X_i \sim \mathrm{Po}\left(\lambda_i\right) \wedge X_i \perp\!\!\!\perp X_j \implies X_i \left| \sum_{j=1}^{n} X_j \right. \sim \mathrm{Bin}\left(\sum_{j=1}^{n} X_j, \frac{\lambda_i}{\sum_{j=1}^{n}\lambda_j}\right)$

Exponential

- $X_i \sim \mathrm{Exp}\left(\beta\right) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_{i=1}^{n} X_i \sim \mathrm{Gamma}\left(n, \beta\right)$
- Memoryless property: $\mathbb{P}\left[X > x + y \mid X > y\right] = \mathbb{P}\left[X > x\right]$

Normal

- $X \sim \mathcal{N}\left(\mu, \sigma^2\right) \implies \left(\frac{X-\mu}{\sigma}\right) \sim \mathcal{N}\left(0, 1\right)$
- $X \sim \mathcal{N}\left(\mu, \sigma^2\right) \wedge Z = aX + b \implies Z \sim \mathcal{N}\left(a\mu + b, a^2\sigma^2\right)$
- $X_i \sim \mathcal{N}\left(\mu_i, \sigma_i^2\right) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_i X_i \sim \mathcal{N}\left(\sum_i \mu_i, \sum_i \sigma_i^2\right)$
- $\mathbb{P}\left[a < X \le b\right] = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$
- $\Phi(-x) = 1 - \Phi(x) \qquad \phi'(x) = -x\phi(x) \qquad \phi''(x) = (x^2-1)\phi(x)$
- Upper quantile of $\mathcal{N}\left(0, 1\right)$: $z_\alpha = \Phi^{-1}(1-\alpha)$

Gamma

- $X \sim \mathrm{Gamma}\left(\alpha, \beta\right) \iff X/\beta \sim \mathrm{Gamma}\left(\alpha, 1\right)$
- $\mathrm{Gamma}\left(\alpha, \beta\right) \sim \sum_{i=1}^{\alpha} \mathrm{Exp}\left(\beta\right)$
- $X_i \sim \mathrm{Gamma}\left(\alpha_i, \beta\right) \wedge X_i \perp\!\!\!\perp X_j \implies \sum_i X_i \sim \mathrm{Gamma}\left(\sum_i \alpha_i, \beta\right)$